Chapter 16
A First Exploration to Understand Mathematics Curricula Implementation: Results, Limitations and Successes

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This chapter is divided into five sections. In the first, Mogens Niss addresses the case of a partially successful curriculum reform in Denmark. It is the KOM Project (that began in 2000) which developed a theoretical proposal that sought to respond to specific difficulties in the transition between various cycles of education including higher education as well as weaknesses in STEM programs. That proposal focused on the concepts of mathematical 'competence' and 'competences', with a perspective that has impacted the international community, especially through its influence on the OECD's PISA tests. To better understand the characteristics of the Danish experience, Niss establishes a much broader conceptual framework that establishes six dimensions in every curricular situation: goals, contents, materials, teaching methods, student activities and assessment. The reform was partially successful as competence-competences were included in the goals of education, in teaching methods and student activities in Denmark, but not, for example, in assessment and content.

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In a second section, Michèle Artigue reports two types of curricular implementation: on the one hand, a successful reform in France at the beginning of the twenty-first century in the senior high school around statistics, the place given to the stochastic domain, the use of computer-simulations, and also the importance of interdisciplinary work. And, on the other hand, it summarises the development of reforms in some Francophone countries or regions: Wallonia-Brussels Federation, Tunisia and Quebec (Canada); some that failed, one crowned with success. In all these cases, strategies, and disciplinary, educational and even social variables are described, which would influence the success or difficulties of these reforms. Artigue analyses these cases by referencing general theoretical concepts of the 'ecological' perspective associated with the Anthropological Theory of the Didactic which is an extension of the Theory of Didactic Transposition.

In a third section, Yiming Cao describes a broad curriculum reform of the teaching of compulsory mathematics in mainland China (which began in 2001 and was finally approved in 2010). Some of the characteristics of the new curriculum are indicated in contrast to the previous one, and its various phases of implementation are discussed.

The fourth section describes various professional development models in the Philippines to try to implement a general curriculum reform approved in 2012. Enriqueta Reston points out the weaknesses of some of them and provides elements of new modalities that, even isolated and sporadic in the Philippines, can support teacher professional development and implementation of this curriculum reform.

In the fifth section, Angel Ruiz synthesises conclusions derived or inspired from the contributions of the preceding sections.

In all these cases, it is possible to observe models of good or inadequate practices, reforms with relatively broad or partial successes, or failures, as well as the role of teacher preparation and support materials for curricular implementation.

National and International Curricular Use of the Competency-Based Danish "KOM" Project – Mogens Niss

This section begins by offering a conceptualisation of 'curriculum' as a vector consisting of six components ('goals', 'content', 'materials', 'forms of teaching', 'student activities' and 'assessment'). So, a curriculum is determined by specifying each of these components and is implemented by enacting them. I present the Danish competence-based KOM Project and discuss the extent to which this project has been implemented in curriculum reforms in Denmark. The answer is 'only partly', primarily because no official measures were instigated to ensure professional development of educational authorities and teachers. Nevertheless, the KOM Project had quite an impact on Danish mathematics education, albeit 'from below',

but ~~had~~ perhaps an even greater impact internationally. Finally, I offer some reflections on the conditions for successful implementation of novel curriculum ideas.

Terminological Clarification of Key Concepts

The title of the panel to which this section is a contribution is: "Implementation of reformed mathematics curricula within and across different contexts and traditions". In addition to 'mathematics', this title contains some key words, such as 'curriculum', 'implementation' and 'reform(ed)', that are in common use around the world, yet carry a lot of different meanings. I therefore find it necessary to begin by proposing some clarification (I hope!) of these and some related terms.

The key word 'curriculum' means rather different things in different places (Niss, 2016). Thus, the Collins Cobuild dictionary (1999) offers the following definition: "A curriculum is all the different courses of study that are taught in a school, college or university" (p. 401). Kilpatrick (1994), in contrast, focuses on a single subject rather than on a collection of subjects and writes, "The curriculum can be seen as an amalgam of goals, content, instruction and materials" (p. 7). A somewhat different definition, focusing on the mathematics teacher and on what is actually happening in the classroom, is put forward by Stein, Remillard and Smith (2007): "we use the term curriculum broadly to include mathematics curriculum materials and textbooks, curriculum goals as intended by the teacher, and the curriculum that is enacted in the classroom" (p. 319; *footnote*).

Irrespective of what definition of curriculum we adhere to, any curriculum is situated and lives within an *educational setting*, i.e. the institutional, structural and organisational entity within which the teaching and learning addressed by the curriculum take place. A prime example of an educational setting is the entire public-school system of a given country. As other examples we may think of a particular school or tertiary institution, or a particular course in, say, a university.

In Niss (2016) I proposed, along the lines of Kilpatrick's definition, to define a (mathematics) curriculum with respect to a given educational setting as *a vector with six components*, as follows:

- *goals* (the overarching purposes, desirable learning outcomes, and specific aims and objectives of the teaching and learning taking place under the auspices of this curriculum);
- *content* (the topic areas, concepts, theories, results, methods, techniques, and procedures dealt with in teaching and learning);
- *materials* (the instructional materials and resources, including textbooks, artefacts, manipulatives, and IT systems employed in teaching and learning);
- *forms of teaching* (the tasks, activities and modes of operation of the teacher in this curriculum);
- *student activities* (the activities of, and the tasks and assignments for, the students taught according to this curriculum);

- *assessment* (the goals, modes, formats and instruments adopted for formative and summative assessment, respectively, in this curriculum).

Specifying a curriculum in a given educational setting then amounts to specifying each of these six components. Furthermore, *implementing* a given curriculum amounts to specifying it, as well as to *carrying it out*, i.e. putting all the six components into practice.

The agency that determines a curriculum and has the power to implement it within some educational setting is the *curriculum authority* for that curriculum (Niss, 2016). It may happen that a curriculum authority chooses to leave some of the six components unspecified. Then these components are open for others, e.g. local governments, institutions or teachers, to decide upon and specify, for instance by way of enactment. In some countries national curriculum authorities specify only a few of the components, typically 'goals', 'content' and 'assessment'.

What, then, do we mean by *reformed* mathematics curricula? The term 'reform' suggests some desired changes of a rather fundamental nature, which are likely to affect several or all components of the curriculum. Usually, one wouldn't use the term "reform" unless at least 'goals' and 'content' are explicitly affected. However, the other components are likely to be affected as well, by derivation, even though this may not be explicitly intended.

Competency-Based Mathematics Curricula: The Case of Denmark

In the late 1990s, the Danish Ministry of Education saw a need for reforming the mathematics (and other) curricula in Denmark across all educational levels. This need was spurred by a number of issues and problems that became more and more manifest and visible within and outside the education system. These included that too many students did not benefit enough from the mathematics instruction they were offered, and that there were serious transition problems and severe academic and socio-cultural discontinuities when students moved from one segment of the education system to the next, from primary through to tertiary education.

These transition problems went hand in hand with insufficient progression in students' mathematical learning within and across these segments, which led to 'consumer' complaints about the decrease in students' mathematical capabilities. Moreover, many people thought that not all teachers were adequately prepared for offering high quality mathematics teaching to their students. These problems were seen as (co-)responsible for the fact that students opted away from further education programmes in science, mathematics and technology, which was (and is) considered a serious societal problem.

Against this background, the Ministry, in 2000, established a commission (a task force), composed of mathematicians and mathematics educators (researchers, teachers, and ministerial inspectors) and a few representatives from society at large.

The Commission was chaired by me, while Tomas Højgaard (Jensen) was its academic secretary. The task of the Commission was: (1) to identify, uncover, chart and analyse the entire set of *problématiques* pertaining to mathematics education at all levels of the Danish education system; (2) to propose measures and tools that were likely to be effective in improving the state-of-affairs by counteracting the problems identified and by remedying (some of) the deficiencies observed; these measures were to include drawing up guidelines for the design of new curricula. The Commission worked for 2 years in what became known as the *KOM Project* ("KOM" is a Danish acronym for "Competences and the Learning of Mathematics"), and ended up publishing a report, known as the KOM Report (Niss & Jensen, 2002; Niss & Højgaard, 2011, 2019), which was discussed widely in several places and quarters in Denmark and soon after in a number of other countries as well (e.g. Germany, Norway, Sweden).

The brief for the KOM Project was far from solely focused on proposing new curricula, but had a much wider scope. So, the Project was *not*, first and foremost, meant to be a curriculum project. However, it was assumed by the Ministry, and also by the members of the Commission, that the design of mathematics curricula could be substantially supported by the outcome of the work. I shall return to this issue below.

The KOM Project took its point of departure in the need for creating and adopting a general conceptualisation of mastery of mathematics that goes across and beyond educational levels and institutions. Only then would it be possible to deal with mathematics in a manner that was neither tied to nor dependent on particular levels and types of institutions, which was necessary in tackling the transition problems in the education system. We also wanted to avoid being locked into the specifics of particular mathematical subject matter domains or topics such as algebra, geometry, functions, calculus etc., the place and content of which vary greatly across levels and institutions.

We therefore decided to base our work on an attempt to define and characterise mathematical competence in an overarching sense that would pertain to and make sense in any mathematical context. Focusing (as a consequence of this approach) first and foremost on the *enactment* of mathematics means attributing a secondary role to *specific* mathematical content, which does not mean that mathematical content as such was to be of secondary importance, of course not.

We came up with the following definition of mathematical competence:

> Possessing *mathematical competence* – mastering mathematics – is an individual's capability and readiness to act appropriately, and in a knowledge-based manner, in situations and contexts that involve actual or potential mathematical challenges of *any kind*.

In order to identify and characterise the fundamental constituents in mathematical competence, we introduced the notion of mathematical competences:

> A *mathematical competency* is an individual's capability and readiness to act appropriately, and in a knowledge-based manner, in situations and contexts that involve *a certain kind* of mathematical challenge.

A metaphor may illuminate the relationship between competenc*e* and a competency: if we think of mathematical competence as a huge, complex molecule (say a polymer), the competences represent much smaller building blocks (atoms or monomers) in this molecule. *Eight competences* were identified, in the beginning on theoretical and experiential grounds only. Later on, they became corroborated empirically. These are:

- mathematical ***thinking*** competency – mastering mathematical modes of thought;
- ***problem-handling*** competency – being able to pose and solve mathematical problems;
- ***modelling*** competency – being able to analyse and construct mathematical models;
- ***reasoning*** competency – being able to reason mathematically in the context of justification of mathematical claims;
- ***representation*** competency – being able to handle different representations of mathematical entities;
- ***symbols and formalism*** competency – being able to handle symbolic language and formal mathematical systems;
- ***communication*** competency – being able to communicate, in with, and about mathematics;
- ***aids and tools*** competence – being able to relate to the material aids and tools for mathematical activity.

Since the competences are meant to go across all mathematical subject matter domains, in a given educational setting it neither makes sense to consider deriving the competences from such domains, nor to consider deriving domains from the competences. Even though the competences can, of course, only be developed and exercised in dealing with subject matter, the relationship between competences and mathematical domains should be perceived as constituted by two independent, yet interrelated dimensions, of a matrix composed of competency rows and topic columns. Each cell in this matrix represents the relationship between the competency in the corresponding row and the topic in the corresponding column. More specifically, it allows one to specify the ways in which this competency plays out in dealing with a given topic, and the ways in which that topic plays out in exerting the competency at issue.

KOM-Referenced Curriculum Reforms in Denmark in the Twenty-First Century

Even though, as mentioned above, the KOM Project was not primarily established as a curriculum project it was certainly intended and expected that the outcomes of the project, including the eight mathematical competences, would be instrumental in designing new curricula that would help counteracting some of the problems

identified prior to and within the project. Although the notion of curriculum introduced at the beginning of this section was not in place at the time of the KOM Project, the project actually adopted a similar notion of curriculum, which was also reflected later in the Danish curriculum reforms of the twenty-first century, in 2009, 2014 and 2017 for primary and lower secondary mathematics (grades K–9) and in 2005, 2013 and 2017 for upper secondary mathematics (grades 10–12). As regards grades K–9, the curriculum reform in 2009 was also much influenced by the report of another committee led by me (*Udvalget til forberedelse af en handlingsplan for matematik i folkeskolen*, 2006), which spelt out ways in which the competence thinking could be implemented in pragmatic terms.

The curriculum documents representing these reforms all included important bits and pieces of the KOM Project but it would be incorrect to say that the reforms were a clear-cut implementation of the Project in its entirety. As to the above-mentioned six curriculum components, these were all addressed in the different curriculum designs, albeit with varying degrees of specification. It follows from what was said above that the 'content' component had to be specified independently from the competences, whereas these contributed to shaping the other components. The 'goals' component, in particular, was typically formulated in competency terms.

In a number of different ways, the KOM Project was a great challenge to traditional conceptualisations of mathematics teaching and learning in Denmark. With the project's primary emphasis on the enactment of mathematics, across education levels and mathematical topics, rather than on mathematical content, curriculum authorities – the official Danish education system, governed by the Ministry of Education, as well as teachers, experienced difficulties in coming to grips with how the outcomes of the KOM Project could in fact guide the design and implementation of new curricula that were not (to be) defined in terms of classical content strands. Furthermore, the issue of assessing competences rather than content knowledge and procedural skills also constituted (and still constitutes) a challenge to the system.

This implied that the new curricula of the first two decades of the century continued to be primarily based on subject matter domains, whereas the competences were presented in the general sections of curriculum documents, stating that the teaching of those domains should pursue competency-oriented goals, whilst paying attention to the competences 'throughout' teaching activities (the vector components 'forms of teaching' and 'student activities').

In Denmark, national exams at the end of grade 9 and again at the end of grades 10, 11 or 12, (depending on which of several possible upper secondary streams the individual student is in) are high stakes exams organised by the Ministry. Without going into details with the somewhat complex exam structure and organisation, the written component of those exams ended up paying almost no attention to the competences. In the oral component, which is mainly dealt with locally within the individual school, there is room for focusing on the mathematical competences, if the teacher so wishes, which is also the case when it comes to formative assessment.

In other words, the crucial curriculum component 'assessment' was never markedly influenced by the competency approach, and since 'what you assess is what you get' this partly jeopardised the competency approach and made it largely

rhetorical at the official level. However, other curriculum components, such as materials (including textbooks), forms of teaching, and student activities were oftentimes influenced by the competency thinking of the KOM Project. The same is true of pre-service teacher education and in-service professional development.

So, whilst the competency approach mainly had a rhetorical impact on the official curricula, especially as regards the components that are somewhat tightly controlled by the Ministry of Education (the vector components 'goals', 'content', and 'assessment'), it would not be correct to say that this approach has had no impact on the implementation of these curricula in everyday practice. As a matter of fact, the competency approach and the associated terminology substantially influence the *discourse* amongst mathematics educators in Denmark, who readily express themselves and explain their activities in terms of the KOM competences.

Ironically, then, we may say that what from the point of view of the Ministry should have provided a top-down platform for an entirely new approach to mathematics teaching and learning never became such a platform, primarily due to inertia in the different segments of the official system; whereas, the approach and the thinking of the KOM Project gradually, in a bottom-up process, crept into significant – but certainly not all – aspects of everyday mathematics education in Denmark. This bottom-up process took several different forms, ranging from a large variety of local implementation projects, typically focusing on a few of the competences at a time (many of which were conducted by the KOM Secretary, Tomas Højgaard), over new KOM-inspired textbook systems for primary, lower secondary or upper secondary school, and expository publications by or for practising teachers, numerous articles in national journals or teacher magazines, through to pre- and in-service programmes and resource materials for teachers. Each of these bottom-up activities typically involved a selection of the enactment-oriented components of the curriculum vector.

This development begs an answer to the question: Why did things happen in this way? Well, this is a highly complex issue, which involves a combination of universal as well as national features of curriculum design and implementation. I shall focus on the national ones.

It is clear that the thinking in and behind the KOM Project and the competency approach taken were highly novel, ambitious and demanding for the Danish education system and for teachers to come to grips with. So, it was far too optimistic to expect that the KOM Project ideas could be transposed into curriculum design and implementation without further ado, just by reading the KOM report. Neither the curriculum authorities nor the teachers asked for, or were given, a systematic, thorough introduction to the ideas and their consequences, or were offered professional development activities beyond the written report itself.

This is typical of Denmark, in which political unwillingness to spend public money on human resources, in combination with anti-elitism, has had a strong foothold over the last 50 years. In retrospect it would have been absolutely necessary for a much more forceful and effective implementation of the competency approach in Danish curricula to have had large-scale, systematic in-service activities within all

layers of the system. In the absence of such activities, the competency ideas had to enter the system mainly by osmosis, which they certainly did.

Against this background it is remarkable that the KOM Project thinking and the competency approach have in fact influenced mathematics teaching and learning in Denmark as much as they have, especially by way of a multitude of enacted versions of the curriculum vector. This can only be explained by the existence of serious needs amongst educational authorities and mathematics educators for conceptual innovation in mathematics education. The policy lessons that can be learnt from this case are primarily two: (1) you cannot effectively pursue goals and aims unless you are willing to invest and apply material and immaterial means that are conducive to the aims and goals; (2) only very rarely are top-down measures successful. If you really want to achieve change, it is essential that those who are to bring it about have ownership not only of the need for change but also of the means to achieve it. If not, you might be able to see changes on the surface of things, but they will not really affect the substance of what is desired and expected.

The Competency Approach in Other Countries

During the first two decades of this century, many countries and quarters took an interest in the KOM Project and in the competency approach to mathematics education (Niss et al., 2016). This was partly, but not exclusively, stimulated by the fact that competency ideas were involved in shaping all the PISA mathematics frameworks between 2000 and 2012 (Niss, 2014) by underpinning and developing the notion(s) of *mathematical literacy*. However, due to direct personal contacts between mathematics educators in Denmark and in countries such as Germany, Norway and Sweden, these countries early on adopted and adapted aspects of a competency approach as well as some of the related KOM Project ideas in their curriculum development. In particular, the German *Länder*, in the first decade of this century, agreed to take an explicit competency approach when reforming their curricula, leading to the so-called 'Bildungsstandards' (see, for example, Kultusministerkonferenz, 2012). Many countries in Latin America and Spain were also inspired by the competency ideas, primarily via PISA.

It is important to observe, here, that it was never a matter of direct translation and adoption into curriculum design and implementation in other countries of the KOM Project ideas or documents. Rather, it was a matter of modification and adaptation of (some of) these ideas so as to suit national circumstances, needs and traditions. Oftentimes, the eight competences of the KOM Project were modified in various ways, typically into fewer than eight. In some cases, adaptations were not even in accordance with 'the spirit' of the Project, only inspired by some of its features.

Once again, there are lessons to be learnt from these developments. Firstly, one should never aspire to directly translating, transferring and adopting curricula or curricular ideas from one country or setting to another. Such import, even of curricula that were highly successful in their original setting, is almost doomed to failure

because the socio-cultural environments and the economic, technological, and institutional boundary conditions vary so much within and across countries. Secondly, the lesson just mentioned should not be taken to suggest that inspiration from others is likely to fail. On the contrary, and this is the second lesson worth highlighting, thoughtful and careful consideration of what others have accomplished, whilst paying attention to the conditions and circumstances under which the accomplishments were achieved, is likely to stimulate positive innovation (and innovation always comes with a "sign" and hence may also be negative) in new places, provided those who are to implement this innovation are genuine shareholders in it.

Implementing Curricular Reforms: A Systemic Challenge – Michèle Artigue

This contribution addresses the challenge of implementing curricular reforms. I first introduce the approach I propose, considering education systems as complex dynamic systems, and the main theoretical elements I rely on, offered by the anthropological theory of the didactic (ATD) and the ecological perspective underlying it. Then I use this approach to discuss the challenge raised by the implementation of curricular reforms using, as case studies, the 2000 high school curricular reform in France, and the implementation of competence-based curricula in three different Francophone countries. I conclude by drawing some lessons from these case studies.

Introduction

As highlighted in the Discussion Document for this ICMI Study, curriculum reforms are transformations that generally affect education systems "as a whole, at a national, state, district or regional level" (2018, p. 572). They modify the conditions and constraints of their functioning to cause changes in the state of these systems. Their *raisons d'être* are situated at different levels: the content of teaching, the balance and relations between school disciplines, pedagogical methods, or more generally the social contract between a society and its schools; more and more these reflect supra-national visions. Their design mobilises a diversity of institutions and agents, and their implementation an even greater number. Design and implementation processes take place over time and their dynamics depend on a multiplicity of factors in interaction.

When a curriculum reform is eventually adopted by authorities, these factors and their possible interactions are only partially identified and even less controlled, if even controllable. The curriculum texts, however constraining they may appear, give some margin of freedom to all those involved in the implementation for expressing their agency, which opens up a range of possible dynamics whose regulation is a crucial issue. In this text, I adopt an *ecological* and *dynamical system*

approach. Within this perspective, questioning the implementation of curriculum reforms and what determines their success or failure, is trying to understand the functioning of such dynamical systems in the face of the *ecological perturbation* that a curriculum reform always is, considering the means used to regulate these dynamics.

Such an ecological perspective being at the heart of the theory of didactic transposition (Chevallard, 1985) and of its extension, the anthropological theory of the didactic (ATD) (Chevallard, 2019), I use these theories to approach the dynamics of curriculum reforms. In the next section, I briefly introduce the main elements of these two theories supporting my reflection.

Elements for an Ecological Approach Supported by the ATD

Didactic Transposition

The theory of didactic transposition was developed in the early 1980s to overcome the limitation of the prevalent vision at the time, seeing in taught knowledge a simple elementarisation of scholarly knowledge. Beyond the well-known succession of transformations of knowledge at the basis of this theory, from scholarly knowledge to the knowledge learned by students (see Chap. 13), ecological concepts such as those of *niche*, *habitat* and *trophic chain* (Artaud, 1997) are also essential in it.

The habitat of a species (here a mathematical object, type of task, technique, …) refers to the environment in which it lives, while its niche refers to the function(s) it has in this habitat. This ecological vision invites us to pay attention to the effect of curriculum reforms on habitats and niches. In addition, it invites us to consider the objects at stake as elements of trophic chains, being fed by some objects while feeding others. Even minor curriculum changes can break trophic chains, and be source of learning difficulties impacting the implementation of reforms. As pointed out in Artigue (2011), this phenomenon is linked to the fact that the official teaching time is distinct from the learning time. The teaching of a new mathematical object is an opportunity for consolidating the relationship with old objects; its zone of influence on learning is an area with fuzzy contours, difficult to identify.

Anthropological Theory of the Didactic (ATD)

The ATD enriches this set of conceptual tools. Key concepts here are those of *institution* and *institutional position* (Chevallard, 2019). Indeed, a curriculum reform mobilises a diversity of institutions for its conception and implementation; it also mobilises agents who occupy different positions in these institutions (the position of teacher is neither that of student, nor that of school principal or parent). To these positions are associated different relationships to mathematical knowledge. Curriculum reforms modify positions and relationships intentionally but also

unintentionally. Understanding these moves and their possible, actual effects, is important for understanding curricular dynamics.

Another essential tool provided by ATD is the notion of *praxeology* used to model mathematical and didactic practices. At its most elementary level, a praxeology (called pinpoint praxeology) is a quadruplet [T/τ/θ/Θ] where T is a type of task, τ a technique or way of processing this task, θ a technology defined as a discourse making this technique intelligible and justifying it, and Θ a theoretical discourse which in turn makes θ intelligible and justifies it. Types of task and techniques constitute the practical block of praxeologies (praxis), while technology and theory constitute their theoretical block (logos). In a given institution, praxeologies do not live in isolation; they are organised into nested structures. Local praxeologies denote groups of pinpoint praxeologies sharing the same technology, while regional praxeologies denote groups of local praxeologies sharing the same theory or piece of theory. Studying the dynamics of praxeological organisations, both mathematical praxeologies and the didactic praxeologies with which they are in dialectic relationship, is a means of gaining an understanding of curriculum dynamics.

As pointed out in Chap. 13, another conceptual tool provided by the ATD is the *hierarchy of levels of didactic co-determinacy.* This tool helps researchers consider the different conditions and constraints shaping curriculum reforms and their dynamics, those internal to the disciplines at stake with the lower levels of the hierarchy, and more general ones with its higher levels (*Pedagogies – Schools – Societies – Civilisations*). At each level different agents intervene, new power relations, new rules of legitimacy are established. These different conceptual tools support the analyses and reflection developed in the next sections.

A First Case Study: The High School 2000 Reform in France

Main Characteristics of the 2000 Reform

This reform of high school general education from grades 10 to 12 offers an interesting case. Not a curricular revolution, it however introduced substantial changes still in effect today. To make clear the challenges posed by its implementation, I briefly describe these changes. For more detail, the reader may refer to Artigue (2003). At the level of school structures, there were no major changes and the three orientations organising the differentiation of teaching from grade 11 (L for literature, ES for economic and social sciences, S for sciences) were maintained. At the pedagogical level, continuity was also evident. The curriculum discourse remained a constructive discourse and the place to be given to problem solving was reaffirmed. But it was also stated that the school institution was challenged by scientific, technological and cultural developments and should regularly rethink its objectives in the light of these developments.

This consideration led to substantial changes. In mathematics, the main ones were the strengthening of the statistic domain with the ambition to introduce grade 10 students to statistical thinking through the experience of sampling fluctuations

with the help of computer simulations, a differentiation according to the L, ES and S orientations more sensitive to their specificities and students' interests with for instance the introduction of graph theory in ES, and an increased emphasis on the interaction between scientific disciplines and more generally on interdisciplinarity, especially with the introduction of interdisciplinary projects called TPE (*Travaux personnels encadrés*) in grade 11.

Due to the change in balance between mathematical domains, trophic chains were cut and new ones had to be created; praxeological organisations had to be built for the new domains introduced as well as their progressive structuring. This was all the more demanding as most teachers had not encountered graph theory or inferential statistics in their academic preparation. Even for those with a university culture in statistics, there was a didactic inversion between the statistics and probability domains, as some introduction to inferential statistics preceded the teaching of probabilities. Moreover, teachers were asked to base the teaching of these domains on study themes selected among those proposed, according to their students' interests, which also required new praxeological reorganisations. Interdisciplinary work, project pedagogy on subjects chosen by students involving the critical use of Internet resources, were also new for most teachers.

The Implementation of the Reform

This reform could have been rejected. Tensions arose between the group in charge of programmes and the General Inspectorate of Mathematics, a key institution for the implementation of curriculum reforms and their evaluation in France. The emphasis on statistics was considered exaggerated by many professionals, especially since it occurred at the expense of other sectors, particularly geometry. Many also wondered about the possibility of making sense of inferential statistics without any probability background, and questioned the sense that students would make of the experimental work based on computer simulations proposed to them. There was also great concern about TPEs, especially among mathematics teachers who wondered whether they would find a role for their discipline in these.

The reform generated vivid and at times hard debates, and the alternation of the political majority in 2002 resulted in some changes. However, globally the reform resisted. The importance given to interdisciplinary projects and modelling, to the stochastic domain, was maintained or even strengthened in the next reform, that of 2010. Several factors undoubtedly made the adaptation of the educational system to this ecological perturbation possible. I list a number of them below, by lowering the levels of didactic co-determinacy. The ambitions of the reform and most of the changes introduced aligned with international perspectives, which contributed to their legitimacy. At the national level, the work carried out by the CREM (*Commission de réflexion sur l'enseignement des mathématiques*), set up at the request of the mathematics community in 1999, contributed to legitimise its global vision (Kahane, 2001).

The reform was carefully prepared by the groups of experts appointed by the CNP (Conseil national des programmes) and bringing together a diversity of expertise. The CNP guidelines ensured coherence at the global level among the disciplines. The expert groups were given 2 years to prepare the programmes, and those of scientific disciplines worked together for instance to ensure that the new introduction of exponential functions as solutions of differential equations would lead to coherent approaches based on the study of radioactivity. This also allowed the creation of new trophic chains involving exponential and logarithmic functions. The groups of experts also produced consistent accompanying documents, covering all new domains and showing how the proposed themes of study could be exploited.

A specific website *Statistix* was created offering teachers the possibility to download dynamic simulations and access statistical data. The IREM network (*Instituts de recherche sur l'enseignement des mathématiques*), an essential actor of in-service teacher education in France, also mobilised, in particular, the inter-IREM Commission on statistics and probability. Locally, IREM groups built situations and progressions, experimented, proposed training sessions and produced a number of paper publications and on-line resources, some in collaboration with the APMEP teacher association. The IREM network and APMEP journals devoted many articles to these innovations. TPE working groups were also created in various IREMs. They supported and analysed the implementation of TPEs in the high schools of their members who were high school teachers, and proposed training sessions based on this experience. French didacticians contributed to these activities. Moreover, which is not frequent in France, a pre-experimentation of TPEs was organised, and when the reform was implemented, its results and a number of tools were made available to teachers by the Ministry of Education. And last, but not least, specific modes of assessment were designed for TPE and the students' marks taken into account at the national examination of Baccalauréat in grade 12.

The collaboration between the different institutional agents acting in different positions at different levels of the hierarchy of co-determinacy that this description shows was certainly crucial for the successful implementation of this reform, making that, retrospectively, it is generally considered as a good reform. Without revolutionising the high school system as is the case for the problematic on-going reform (see Arnoux, 2018), it succeeded in introducing important changes. This story also confirms that the dynamics of a reform is a long-term process, the implementation of a new curriculum being just a step in a process whose stabilisation requires many years.

In the case of this particular reform, after a few years, it was observed that the training demand in the teacher community regarding the teaching of graph theory, the TPE and the new praxeological organisation proposed for calculus, strongly decreased, making clear that the system was reaching some stable state. The process was much longer for the teaching of inferential statistics. Moreover, reaching a stable state does not mean that there are no more problems, that the intended and implemented curricula are fully aligned. For instance, still today finding a niche for mathematics in interdisciplinary projects is challenging for many teachers.

A Second Case Study: Recent Curriculum Reforms in the Francophone Space

In 2012, as part of the EMF conference in Geneva, two round tables were organised on how recent curriculum reforms were designed and implemented in French-speaking countries. Six countries or regions were considered: the Wallonia–Brussels Federation in Belgium, Burkina Faso, Quebec in Canada, France, Romandy in Switzerland and Tunisia. The round tables were prepared by a 2-year collaborative work. The perspective adopted was to conceive curriculum reforms as changes in the social contract between school and society, at a time when the tercentenary of the birth of Jean-Jacques Rousseau was being celebrated in Geneva.

The work carried out considered recent curriculum reforms from their conception to their implementation, specifying the educational and curricular contexts, identifying the institutions involved in the reforms and their respective roles, describing the global curriculum dynamics, before focusing on a dimension particularly important in each case study. In this text, I focus on the implementation of reforms, and due to space limitations, I just briefly contrast three case studies, regarding respectively the Wallonia-Brussels Federation in Belgium, Quebec and Tunisia. Detailed analyses are accessible in the 150 pages of the section of the EMF proceedings devoted to these round tables.[1]

The reason for this selection is that the three case studies share one characteristic of particular interest for this ICMI study (see the Discussion Document): They correspond to curriculum reforms proposing a global reorganisation of the curriculum around the concept of competence. As was the case in Denmark (see the first section in this chapter), in the three cases this move towards competences started nearly two decades ago (in 1997 in Belgium, in 1995 in Quebec and with the 2002 reform in Tunisia). In the three cases also, we observe a proximity of the global aims of these reforms in terms of adaptation to a rapidly changing world and to the technological evolution, of increasing learning opportunities for all students and inclusiveness, development of students' learning autonomy and citizenship. These shared characteristics clearly show the influence on these reforms of conditions situated at the highest levels of the hierarchy of didactic co-determinacy, not specific of a given society.

However, and also one important reason for this selection, the case studies show three different dynamics with very different outcomes. In the case of the Wallonia-Brussels Federation, the analysis provided is rather critical. The co-authors (Baeten & Schneider, 2012) relate the difficulties met to three main factors: first, the fact that the reform went along with a policy of centralisation and increased control of the education system with the creation of assessment tools to serve as external references common to the three education networks existing in the region; second, the emphasis put on transversal competences, valid for all disciplines, expressed in

[1] See round tables in plenary activities at: http://www.emf2012.unige.ch/index.php/actes-emf-2012

very general terms and poorly coordinated with the mathematics content that remained nearly the same; third, limited resources and training for the teachers, not addressing their real needs. Training sessions did not help them to create a new praxeological coherence intertwining competences and mathematical content. Ten years after the implementation of the reform, a very critical report led to substantial revision and a regaining of attention to the specificities of the disciplines while trying to avoid disciplinary compartmentalisation.

In the case of Tunisia (Smida et al., 2012), also, the analysis is critical, but the dynamics is different. The conception of the 2002 reform obeys a new institutional organisation involving three different commissions: a first commission responsible for defining the aims of the education system and preparing specifications for the disciplinary commissions and for setting curriculum structures (something analogous to the French CNP mentioned in the first case study), multidisciplinary commissions (science, languages, humanities, art) in charge of delimiting transversal competences and, finally, disciplinary commissions in charge of writing the programmes, taking into account these competences. Visibly, this structure and also the careful analysis of a selection of foreign programmes by the mathematics commission made it possible to avoid the disconnection between competences and content observed in Belgium.

In the Tunisian case, the difficulties observed mainly situated in the implementation phase, a top-down process under the full responsibility of the Inspectorate, carried out with very limited resources. According to the authors, the negative effect of these conditions situated at the school level were aggravated by two factors: the coincidence of the reform with a policy of decentralisation and the heterogeneity of the body of inspectors whose number had tripled in 5 years. Despite the careful preparation of the reform, these conditions of implementation, the lack of clear training strategies and resources, the importance of the changes expected both in terms of mathematics and didactic praxeologies, led to significant resistance among teachers. The specific study the authors conduct on the algebra curriculum illustrates this very well. Moreover, the analysis of teaching practices carried out in Ben Nejma's (2009) doctoral thesis tends to show that some years after the implementation of the reform, the implemented curriculum in algebra was still a mixture of old and new curricula.

The case of Quebec (Bednarz et al., 2012) contrasts with the two first cases. The authors show a long process of curriculum development beginning with the 'États généraux sur la qualité de l'éducation' in 1995 and ending in 2008. This process co-ordinates the action of a multiplicity of actors, coming from various horizons, and clearly rejects the 'top-down' logic that had prevailed until then. More specifically with regard to implementation, some interesting characteristics can be highlighted:

- large-scale implementation was prepared by previous work in pilot schools with support in context, responding to local needs and ensuring that each school developed its expertise and autonomy;

- implementation was supported during more than a decade by substantial training activities both at national and regional levels. National activities targeted educational advisers, resource persons and managers, and focused on the global elements at the heart of the reform such as the concept of competence. Disciplinary issues were addressed at regional level, targeting teachers and pedagogical advisors. In mathematics, the emphasis was placed on the concrete construction of situations by teachers, with as much as possible experimentation of the situations collectively designed in classrooms and *a posteriori* joint analysis.

In addition, a permanent process of regulation was planned by the Commission des États Généraux. A specific commission to which a mission of continuous regulation was entrusted was officially established by the Minister of Education in 1997 and it worked until the end of its mandate in 2010.

This case shows thus a coherent global process of design, implementation and regulation, conceived as a continuous process obeying a participatory logic, and combining top-down and bottom-up dimensions. The evolution towards a curriculum structured in terms of competences took place in this context. Accompanying and regulation work was required, but the move towards competences was not reconsidered. As the authors point out in the conclusion of their study, what the case of Quebec shows is the case of a curriculum that is constantly developing, a 'living' curriculum that leaves room for teachers and other school stakeholders to make it their own. This is a demanding but visibly productive vision.

More globally, this second case study shows that the move towards curricula organised around the idea of mathematical competence or mathematical competences is a major ecological perturbation. Normally, such a move should lead to reconstructing on other bases the existing praxeological organisations and the associated learning trajectories, and to ensuring the viability of these reconstructions. This can only be a long-term process which, given the uncertainty of its dynamics, must be firmly supported over time and regulated. If it is to succeed, it must also obtain and maintain the adhesion of the various actors, particularly the teachers, organise and support their collaboration. The examples described above, like that of Denmark, show that these conditions are far from always being met for different reasons, and the resulting problems of implementation.

Concluding Comments

In this text, I have adopted an ecological and dynamic perspective to approach curricular reforms, relying on constructs provided by the ATD to support this reflection. What lessons can be drawn from this reflection? First, the case studies briefly reported clearly show that recent curriculum reforms express rather close visions of what our societies expect from mathematics education. Common trends are observed, such as the move towards curricula structured around competences, the increased importance attached to showing the role of mathematics for addressing

societal and environmental issues, to the connection between STEM disciplines and to interdisciplinary practices, the increasing space given to the stochastic domain, and the attention paid to students' specific interests, abilities and needs. These confirm that conditions and constraints situated at the highest levels of the hierarchy of didactic co-determinacy influence these reforms.

However, these case studies also show the specificities of each context and the diversity of the resulting curricular dynamics. They make clear that, no matter how carefully a curriculum reform is designed, the dynamics it generates remains partly unpredictable. The vision of curriculum reforms as ecological perturbations and the systemic approach used help understand this unpredictability and also why the information we can gain from pre-experimentations is necessary limited: their experimental status means that they take place in ecologically protected environments. So, the success of a curriculum reform significantly depends on the strategies developed for its implementation, and on the quality of its regulatory mechanisms.

Another clear lesson is that implementation must be conceived as a long-term process, and not something limited to a few years; that long-term support must be provided to all those involved in the implementation and especially to teachers; that the production and accessibility of appropriate resources, the combination of top-down and bottom-up processes, are crucial conditions. The Quebec case study seems a good illustration and the vision of 'living curriculum' a promising one. However, it seems that too often most of the efforts are still focused on the design of reforms, much less on their implementation, monitoring over time and regulation, leading to abrupt changes and ecological disruptions highly damaging for education systems.

In this section, the success and failure of reforms have been discussed. But how are these evaluated, how can they be evaluated? A variety of criteria are undoubtedly to be considered. In the first case study, we mentioned, as criteria of success, the resistance of the reform and that of the main transformations it had brought beyond the reform itself. This is a sort of minimal criterion. It shows that the reform has succeeded in creating a certain level of ownership, which can also be studied from other sources, such as publications and debates generated by the reform. However, as has been pointed out, this does not guarantee the satisfaction of another essential criterion, the alignment of the implemented curriculum with the intended curriculum.

For this, other evaluation instruments are necessary. They may relate to the resources used by teachers and those they produce; the assessments they draw up or the examination papers which are known to strongly condition their practices; they may also be specific surveys, supplemented by observations of real practices. However, the success of a reform can also be appreciated through the way in which the distance between the intended and implemented curriculum, and the feedback from the different actors, are taken into account to regulate it, in the spirit of living curriculum mentioned above. Finally, any reform aims at improving student learning, and a third level of evaluation situates at the level of the achieved curriculum. This requires that the assessment instruments used be aligned with the spirit of the reform and its precise expectations. Unfortunately, the desire to compare the before and after of the reforms too often overlooks this necessary condition.

Chinese Mathematics Curriculum Reform in the Twenty-First Century – Yiming Cao

Curriculum reform is a fundamental factor in pushing forward educational development. In this chapter, I examine the development and implementation of Chinese mathematics curriculum standards. My goal is to present to the world the current situation of mathematics curriculum reform and development in mainland China (i.e. China, excluding Hong Kong, Macao and Taiwan) since 2000.

The Background of New Century Chinese Mathematics Curriculum Reform

Social and economic development in China (especially the development of information technology, digital technology, life-long learning, and democratisation – The Research Group of Mathematics Curriculum Standard, 2002) have raised the bar for mathematics literacy. New demands for modern citizens have required corresponding changes in public schools, especially in mathematics curriculum and instruction (Ma, 2001). From June 1996 to 1997, the division of basic education in the Ministry of Education organised a survey to investigate the status of the implementation of compulsory education in all subjects, including mathematics, across the nation. The data and facts collected from this survey demonstrated that the curriculum used at that time achieved certain goals (e.g. basic knowledge and basic skills training); however, many problems were identified. At the same time, teachers struggled with students having many problems (Liu, 2009). The old curriculum was highly centralised, with little flexibility for local adaption, and it did not meet the different social and economic requirements of a diverse student body. The trends in international and national education that were mentioned above demanded curriculum reform. Similar to the previous education reforms, the current one adopted a top-down approach: however, we cannot negate the fact that it also reflected certain concerns raised from the community.

Mathematics Curriculum for Compulsory Education (Grades 1–9)

The Development of a New Standard for Compulsory Education

The Mathematics Curriculum Standards for Full-time Compulsory Education (draft) (MCSFCE) was completed and put forth for extensive comments from the community in March of 2000. The development of the mathematics curriculum played an important role in this round of curriculum reform in fundamental education, which provides the idea of basic value, the mechanism of implementation, and

the way to develop the standard for other subjects in fundamental education. The Ministry of Education formally promulgated and implemented Mathematics Curriculum Standards for Full-time Compulsory Education (Trial version) in June 2001.

The Implementation of Standards for Compulsory Education

Before the release of the MCSFCE a set of textbooks based on the idea of the new curriculum had been designed by a research group for experimental use (the majority of the members were to part in the later development work of the MCSFCE). Since 1994, this group had conducted two rounds of experiments; more than 60,000 students from more than ten provinces (including both well-developed school districts and undeveloped school districts) participated, which provided abundant empirical experience for the later implementation of the MCSFCE.

The Ministry of Education started a national curriculum reform conference to convene the implementation of the new curriculum in July 2001. Several decisions were made at the conference. First, the overall objectives and strategies for the implementation of the new curriculum in public schools were determined. Second, the strategies to spread the curriculum reform to all Chinese public schools were developed. Third, professional development and teacher training programs were set up. The positioning of the trial version of the curriculum standards necessitated a multi-stage process for spreading the new curriculum. The first stage was to set up the goals, then to conduct preliminary experiments before the nationwide implementation, and finally to broaden the experiment gradually.

In the initial round of experimental implementation of the curriculum, school participants were recruited on a county basis, in 2001. First, applications to be volunteer schools were submitted by counties and were examined before being approved by the Ministry of Education. Forty-two regions (3300 elementary schools, 400 secondary schools) participated in the first round of the national curriculum reform with about 270,000 first graders (1% of the population of first graders nationwide) and about 110,000 seventh-grade participants (0.5% of seventh graders) in 2001. Starting in 2002, each province developed a curriculum reform plan at the province level and determined their experimental regions. There was a total of 570 experimental regions with 20% of Chinese first graders and 18% of the seventh graders participating in the new curriculum.

Subsequently, more schools from an additional 1072 counties became experimental regions at the province level, bringing in about 40–50% of the student population of each grade. Including the earlier participants in 2001 and 2002, there were 1642 experimental regions with about 35,000,000 students participating in the new curriculum in 2003. Based on the results of these pilot tests, the new curriculum entered the phase of nationwide promotion. By 2004, 90% of the school districts in

China were using the new curriculum. As of 2005, except for a few places, the new curriculum had been implemented all over mainland China (Ma, 2009).

Epilogue

In the past 10 years of curriculum reforms, including the Mathematics Curriculum Standards for Full-time Compulsory Education (draft) or Mathematics Curriculum Standards for Compulsory Education (2011 Version), the fundamental research was far from enough. In fact, the existing output of research in primary and secondary school education in the Chinese context was too little to allow for the shaping of a persuasive, rational and substantial data-based curriculum standard. But this lack of sufficient research was not a reason to delay. It was an exploratory process which needed to be refined and improved continuously. The curriculum was expected to have different functions. As the curriculum promoter, the government needed to participate in the academic arguments.

The path of reform was an exploratory process. It was necessary to synthesise theory and practice from mathematics, education, psychology and many other disciplines, pooling resources from all areas and levels, from the most academically high-achieving to the rural schools. The success of the curriculum reform demands rigorous academic attitudes, national responsibility and steady work.

Implementing the *K to 12* Mathematics Curriculum in the Philippines: Models and Processes of Teacher Development – Enriqueta Reston

The *K to 12 Basic Education programme* in the Philippines in 2012 is a major reform that posed challenges in closing implementation gaps through more responsive and sustained teacher development programmes. In particular, the intended K to 12 mathematics curriculum adopts a spiral progression approach where five learning domains; namely: numbers and number sense, measurement, geometry, algebra and patterns, and statistics and probability cut across the grade levels with increasing complexity. With the goals of developing students' critical thinking and problem-solving skills, school mathematics teachers are confronted with various challenges as key implementers of the reform. This paper examined the models and processes for professional teacher development that have been carried out in the Philippines to address the needs for school mathematics teachers in expanding their knowledge bases and enhancing their capacities for implementing the reformed mathematics curriculum.

The Philippine Educational System and the Contextual Realities of Curriculum Reform

In 2012, the Department of Education launched the *K to 12* Basic Education program which is a major curriculum reform in the educational landscape of the country aimed at expanding the basic education cycle from 10 to 12 years and, at the same time, enhancing the quality of educational outcomes (DoE, 2012). From a national perspective, this educational reform primarily reflects the shared experience of change of a country's educational system as it adopts to changing contextual realities of the twenty-first century, national priorities and emerging global standards.

The K to 12 Mathematics Curriculum Reform Mathematics Curriculum Reforms in the Philippines

Some salient features of the reformed curriculum which has substantial impact on the teaching of Mathematics and Science include the use of a spiral progression approach to ensure mastery of knowledge and skills at each level and the use of pedagogical approaches that are constructivist, inquiry-based, reflective, collaborative and integrative (DoE, 2012). These features have profound implications on the training of both preservice and in-service mathematics teachers.

The intended *K to 12* mathematics curriculum encompasses five learning domains with the development of problem solving and critical thinking as the twin goals of mathematics teaching, Inspired by Bruner's model of the spiral curriculum, the adoption of the spiral progression approach to curriculum design in the *K to 12* Mathematics curriculum implies that the same concepts are developed and taught from one grade level to the next in increasing complexity and sophistication (Tan, 2012).

Professional Development Models and Processes for K to 12 Mathematics Teachers

The challenge of closing curriculum implementation gaps lies in the hands of the teachers who are the key actors in any curriculum reform Leung (2008). Different stakeholders of Philippine education from both government and private sectors responded to this need for teacher development in response to the reform. The Department of Education (DoE) conducts annual National Training of Trainers (NTOT) among selected teachers by year level and subject areas to build their capacity as teacher-trainers who will conduct the mass trainings by geographical regions and by academic subjects (DoE, 2016). This is an application of the *Cascading Model* where in-service trainings and seminars move from the national,

regional, division, then school level with decreasing duration at each lower level (Bentillo et al., cited in Lomibao, 2016). These in-service trainings and seminars usually span for 2–5 days and conducted twice a year, during midyear break and summer break.

Another model of professional development is the *Cluster-based training* which involves teachers from several schools attending the same training program conducted by invited subject specialists as trainers with the content determined by the master teachers and the department co-ordinator of the schools in consultation with the teachers (Ulep, 2006).

More progressive models of teacher development have been explored to address professional development needs of mathematics teachers. The University of the Philippines National Institute for Science and Mathematics Education Development (UPNISMED) has advocated and used the Lesson Study approach for science and mathematics teacher development in various schools within Metro Manila and nearby provinces (UPNISMED, 2017). The Department of Education (DoE, 2016) also institutionalised the Learning Action Cell (LAC) as a school-based continuing professional development strategy where groups of teachers engage in collaborative learning sessions to solve shared challenges encountered in the school facilitated by the school head or a designated LAC leader. The LAC shared some commonalities with lesson study as it promotes teacher collaboration and the growth of professional learning communities or school-based communities of practice, though there are marked differences in focus of the collaborative learning sessions and group structure.

A Needs-Based Professional Development Model for K to 12 Mathematics Teachers

In response to the challenges of the *K to12* reform, the Science and Mathematics Education Department of the University of San Carlos conducted a needs assessment survey in 2015 for Mathematics teachers from public and private schools in Metro Cebu, Philippines. The results revealed that *Probability and Statistics* was ranked 1st by majority of the teachers as the area where they are least confident to teach and in which they need more professional development (Reston & Canizares, 2019).

Based on the need assessment, a 5-year teacher development project entitled *Improving Statistics and Probability among K to 12 Mathematics Teachers in the Philippines* was launched in 2015. The project was implemented in three phases which included: (1) capacity building of workshop facilitators along with the development of activities and learning resources for the workshops; (2) the conduct workshops in parallel sessions for elementary, junior and senior high school mathematics teachers; (3) the development of an online support structure for participating teachers to access additional resources, share best practices and participate in a professional learning community of teachers.

Implications and Future Directions

The evolution of professional teacher development models to address teacher needs in implementing the reformed mathematics curriculum is indicative of the openness and flexibility of various institutions and professional teacher groups to embrace a wide range of options to improve teaching quality and learning outcomes. While the training model may be efficient as it offers a wider reach to the greatest number of teachers in least time, the more progressive and transformative models have provided teachers opportunities for involvement, collaborations and reflections into one's professional development and teaching practice. Evaluation of the impact of these programs are needed to inform both educational practice and research in mathematics teacher development.

Implementations of Reforms Are Diverse, Multifactorial and Non-linear – Angel Ruiz

The implementation of curriculum reforms cannot be viewed as a linear process. There are always ups and downs, and inflexion points. The nature of the reform conditions very much the implementation, but the strategies that are adopted and the educational agents that participate also do. One of the reasons for non-linear developments is that there is always debate and struggle within diverse contexts. Besides, the factors that intervene within reforms implementation are not only multiple, but the weight of each factor is different. Diversity, again, should be emphasised. The aim of this final section is to collect and contrast some aspects that emerge from the reforms introduced so far.

The Nature of the Reforms Studied

In all these experiences, the nature of the reforms is inscribed in strong international trends, such as the cultivation of competence or twenty-first-century skills; here a 'pragmatism' in relation to mathematical preparation usually includes the aim to serve individual and collective progress and a lifelong education perspective. That is why abilities and not only contents are invoked.

These international influences impacted the reforms described on mainland China, although it should also be added that there were some important local inputs: The previous curriculum was questioned as old, complicated, difficult, with an emphasis on memorisation, repetition, rote, and too centralised to allow proper implementation.

The same is true of the Philippines where the so-called *K to 12 Mathematics Curriculum* also insists on achieving mastery of knowledge and skills at each level and the use of pedagogical approaches that are constructivist, inquiry-based,

reflective, collaborative and integrative. To support the French reform we have reported, international trends were used that promote the role of statistics (stochastics), the place of technologies and the strengthening of interdisciplinary initiatives in education (especially STEM).

The 'competence' and the 'competences' and other results elaborated by the KOM Project became one of the crucial nutrients in the theoretical framework of the PISA tests, which has impacted curriculum reforms in many parts of the world. Although these reforms have used or even nurtured general international trends, all responded to local precise needs, different for each one.

Some Particular Aspects

In the implementation of the reform reported in France, one feature we can point out is the building of a crucial convergence between different important educational agents (associated with national authorities, scientific or teaching communities). The case of mainland China shows a carefully managed implementation process in various phases: A trial proposal by a group of experts appointed by the Ministry of Education, which was submitted for consultation in provinces and experimental schools, and that included a very early revision process. The final version was finally published in 2010. We can observe in the Philippines a movement towards the exploration of training models more focused on specific environments, in which there can be greater interaction, collaboration, reflection, what seems to be an international trend. What struck us about the Danish reform it is how educational authorities included the perspective proposed by KOM in some of the curricular components, but not in other key ones (no official implementation processes were created following KOM's ideas, training or elaboration of materials or national assessment).

The Dichotomy Between Top-Down or Bottom-Up Strategies, and Other Factors

Diversity and multifactorial developments can be seen through the prevalence of a general factor of reforms: The role and weight given to top-down strategies or to bottom-up actions.

In the case of the Wallonia-Brussels Federation, a clearly very dominant top-down reform was formulated. The balance after 10 years is not reported as positive. One factor is also indicated: The reform did not take into account the specific disciplines (in this case mathematics) and it tried to force a general framework of competences that had to be applied at all levels and that also included external evaluation mechanisms; with an emphasis placed on a transversal vision. Coordination with mathematics was non-existent. An equilibrium between general and specific

constructs was not developed. And another aspect: Very few materials and limited training were provided to support teachers in implementing the reform.

The case of the Tunisia reform was also basically a top-down process, but the same mistakes as described in the Belgian Francophone case were not made here: The 2002 reform avoided the drastic separation between a general competence approach and mathematics. But that was not enough to achieve success. They were unable to provide enough support materials for teachers and there were no clear strategies for their training. This factor was crucial. But another aspect was pointed out: Given general policies of institutional decentralisation (external to mathematics), there was a great heterogeneity of educational inspectors. In this scenario, teachers have resisted implementing this reform.

In Quebec, the reform process studied had an incubation and development phase between 1995 and 2008. Unlike what happened in Francophone Belgium and Tunisia, a process that did not have a top-down dominant orientation was sought. The participation of pilot schools was taken into account, seeking to develop their expertise and autonomy. Then, for more than 10 years, training processes were given at the national and regional level on the competence approach. The properly disciplinary subjects (mathematics) were given at the regional level. The trainings focused on teachers designing problems or problem situations that were later socialised. We have a report on a harmonious combination of factors: An equilibrium between top-down and bottom-up actions and general and specific reform constructs, and teacher's sound participation in the designing of pedagogical resources.

A bottom-up orientation offers important results in terms of appropriation of the reform by educational agents (especially teachers). However, to properly develop this type of strategy requires time, resources and certain quality of these educational agents. And in some contexts, such as in developing countries or regions, it is not easy to have the time to implement a reform, as they often depend on unstable political support and socio-cultural understanding and maturity. It is common that the demands for immediate results (measured for example by better student performance or school promotions) are in these contexts very persistent. Similarly, weaknesses in the quality of teacher preparation (pre-service and in-service) and resources can have significant effects. But additionally, in 'very ample and participative' processes there is a risk that the characteristics of the reforms will be distorted if they are not conducted properly.

On the other hand, top-down strategies can have different levels of success or failure that can be due to other variables. The case of French-speaking Belgium shows us, indeed, a top-down but poorly formulated orientation and mistakes. The reform in China, on the other hand, can be placed as a top-down orientation but where the participation of the provinces and teachers in schools throughout the process is reported. The case of Tunisia shows us another experience better directed from the top but that was not able to articulate resources, training and a positive commitment of teachers. In the French case we see a consistency-convergence between various educational agents (bottom) and politics (top). Promoting bottom-up strategies in reforms is necessary, but not enough. Other factors and the combination of them do impact reform's outcomes.

The reasonable recommendation is what Artigue underlines: To achieve a harmonious combination of top-down and bottom-up strategies. Something that should draw on the international good practices or lessons, but also that requires careful calibration of the specific contexts in which a reform it is to be implemented.

Struggle and Uncertainty

In the French case, there were confrontations of ideas between various professionals and groups, but that did not prevent the reform from continuing. Our reading tells us something obvious: The existence of groups of reformers that are struggling for these changes (and they indeed succeeded). The reforms are processes that have names of individuals or groups, something that is sometimes diluted in institutional anonymity. But here there was another element: There was not always certainty that the reform was going to take place and remain.

In Denmark, reformers (around the KOM Project) show the case of a group of experts that failed to gain national political support in their country to help sustain a new curriculum in all dimensions, but other reasons they had a strong impact outside their country (even in governments, and in other very political international institutions).

The 'ecological' theoretical approach that Artigue uses aids us to better understand this uncertainty in particular by highlighting how a reform impacts many dimensions of education and society, of which we cannot even be aware, and less so in advance. Besides, the characteristics of the local groups within social, cultural, or political scenarios impact the certainty or uncertainty of the implementation of a reform.

Final Comments

The Quebec experience shows us a reform following gradual steps with broad participation. Both this case and that of France occurred in advanced socio-economic, educational and cultural contexts. In China, a controlled and organised reform process within a very stable political context is reported. In all these cases, without a doubt, good materials, extensive training, infrastructure, the supportive participation of professional groups or associations or educational officials, and reasonably good quality teachers have been crucial. In all these cases the process took no less than 10 years. And in the developing world? We believe this would be more difficult to replicate in developing countries or regions; not only because of the weakness in available material or human resources, but also because social and political instabilities and uncertainty tend to outweigh here more than in other contexts.

In the cases studied so far, there are factors that we find relevant in the development of a curriculum implementation, clearly teacher preparation and adhesion to

the reform, and resources. But other dimensions also weigh in, such as the role of more general educational agents (such as inspectors in Tunisia) whose participation depends on institutional policies, or the "maturity" of the mathematics and mathematics education communities to help implement the changes; or the legitimacy and consistency of a reform (something that in the French example academic communities were instrumental in achieving). The role of these different factors within each implementation process and the way they get articulated provoke each context to have diverse outcomes.

We have seen success, failure, limitations and positive developments in these reforms. How should such reforms be monitored or evaluated? One first general approach may be using the six categories that Niss offered us: to gauge how successful a curricular reform or proposal has been by calibrating what happens in each of the vectors indicated, and of all of them as a whole. Of course, the "survival" in the time of a reform is a parameter, especially when there is deep struggle in the educational communities. The adhesion to or rejection of the reform by teachers can be another criterion. Artigue points out what can be another criterion: When teachers have reached a level of mastery such that they do not require further training (mostly). Cao points out (and Artigue does too) commonly used mechanisms to gauge understanding or support for reform: Surveys, situation analysis and discussions of special issues.

Reform development assessment is not easy in part due to many variables that intervene, and also because they usually require a long-term where many things can happen in different order. As we said before, curriculum implementation does not follow a linear path, and it is complex to determine, for example, in which point of the path does a reform stand. Within this discussion, it seems wise to underline as a criterion to gauge reform implementation the *alignment* of the reformed curriculum in relation to the different means (resources, teacher training, ICT technology role) developed for implementation, and in particular the quality, adequacy and up to date of these means in connection to the international experience and research. However, for political authorities and the general populations it becomes sometimes difficult to acknowledge the progress of a reform leaning only on such elements, there are a collective demand for 'visible' results in the shortest time. And furthermore, a long-term perspective is more difficult to achieve within developing contexts. This may impact negatively the success of 'good' reforms.

Finally, the curricular vectors that Niss describes in this chapter help us to visualise curriculum implementation not as an indivisible amorphous whole but as a process with various components that can be developed in different ways, and between which there may or may not be consistency. However, one more variable could be added in relation to these vectors, time (or timing). It is possible when evaluating a reform that the goals and content can be stablished in a moment but that forms of teaching and student study develop much later, or even that the assessment component can be postponed even more to allow for other components to advance. The concrete analysis of the specific situation is invoked. This reinforces the recommendation not to extrapolate, since what works in one context may not work in another.

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