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Plenary Panel 1

Actors for Math Teacher Education: Joint Actions versus Conflicts

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ABSTRACT We focus on the interaction of mathematicians and mathematics educators as they relate to the preparation of prospecting teachers and professional development of practicing teachers. We emphasize collaborative experiences and show how much can be gained with close collaborations. For this, we describe some examples and point out various factors that have made collaboration possible as well as potential conflicts that existed in certain institutional, cultural political and social-economic environments, and thus draw emerging issues. It is not intended to state rigid conclusions applicable in all contexts, however we consider two general perspectives and suggest some questions to guide research on this area. In general, although we did not find systematic strong research on the collaboration of mathematicians and mathematics educators in the context of teacher professional development, this does not necessarily mean that such efforts do not exist in different countries.

Keywords: Actors; Communities of practice; Conflicts; Cooperation; Mathematics educators; Mathematics teacher education; Mathematicians.

1. Introduction: Joint Actions versus Conflict — Some Key Characteristics

Various actors, with different roles and impact, interact as they participate in mathematics teacher education. These actors can be, for example: mathematicians, mathematics educator researchers, pedagogues, mathematics teacher educators, teachers, mentors, policy makers, curriculum developers, heads of schools, administrators, parents, or students. The actions of these actors and their impact on

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mathematics teacher education depend on several factors such as for example the institutional and social contexts where the interaction takes place or the availability of resources.

The role of these actors and these factors have varied over time. They were not the same for example during -let's say- the "New Math" and "Going to Basics" during the sixties or seventies of the previous century, or during the recent decades where "Competencies" and "XXIst Century Skills" are promoted.

If we look at these roles today, the conditions and the actors involved in mathematics teacher education have evolved in many settings. Some of these changes are due to research in mathematics education (during the last two decades) that has focused on the mathematics teacher and has contributed to the development of teacher education practices that seem to be crucial in the education of both prospective and practicing teachers. For example, the focus on the special nature of teacher knowledge has an impact on the courses offered to prospective teachers and to the professional development of teachers (Cooper and Karsenty, 2008) where mathematics teacher educators and mathematicians play the central role. The emergence of practice-based pedagogies emphasized the importance of field experiences in teacher preparation and professional development (Solomon et al., 2017) that involves mainly mathematics teacher educators, mentors, teachers, administrators. Currently, research on large-scale studies on professional development has considered the important role of policymakers and has also developed strategies that involve a large number of teachers (Maass et al., 2019). In this setting, besides the policymakers, mathematics teacher educators, mathematicians, teachers are also key actors. This research has played an important role and attempts have been made to become available to other actors such as mathematicians (an example of this is the papers on solid findings written by the Education Committee of the European Mathematical Society (EMS) in the newsletter of EMS, see e.g. Hoyles, 2014), to teachers (teacher journals, professional development activities, conferences), to policy makers (through conferences and policy makers workshops, e.g. in the conference Educating the Educators III organised by the International Centre for STEM Education in Freiburg, Germany, https://icse.eu/educating-the-educators-iii/). Moreover, the emergence of international comparison and ranking of nations in math performance for students (PISA, TIMSS) and the last years for teachers (TEDS study) is also one aspect of the international context which has an impact on how the mathematical preparation of mathematics teachers is perceived.

The actors that we have mentioned above who play a crucial role in mathematics teacher education, as well as the underlying factors that drive their actions, have intervened and intervene in teacher education in different ways, in diverse cultural contexts (e.g. European, Asian, African) and socio-economic contexts (e.g. developed world, developing world, countries in transition).

Although each community of actors usually has its own goals and perspectives for the preparation and professional development of mathematics teachers, the collaboration between them is essential for promoting an effective way of intervention.

The attention in section 2 is given to the interaction of mathematicians and mathematics educators in connection to prospective teachers' preparation and, in section 3, to the professional development of practicing teachers. The fourth section is devoted to collaboration as a Community of Practice.

2. Mathematicians, Mathematics Educators, and Teachers in the Initial Teacher Education

2.1. Collaboration and conflicts between mathematicians and mathematics educators

The initial teacher education for prospective secondary school mathematics teachers includes several actors, and often varies according to the targeted level (grades K-12) of teaching and the presence of specialist mathematics teachers (as opposed to generalists). The level of involvement of mathematicians and mathematics educators and the potential for joint action, collaboration and conflict is dependent on context and varies greatly between countries.

In some settings, the potential for joint action appears to be lower, as mathematicians and mathematics teacher educators (MTE) participate, essentially separately. This may be the case when prospective secondary school teachers spend most of their university education in mathematics courses taught by mathematicians. In some such settings, a broad body of research has documented a disconnection between the mathematics taught and practiced at university and the mathematics required for school teaching (Zazkis and Leikin, 2010). While some mathematicians have looked at the differences between these type of mathematics in a positive light and many have acted to address this, there are tensions which can lead to conflict, as illustrated by the Math Wars (Ralston, 2004). To develop connected knowledge, numerous collaborative efforts between mathematicians and MTEs have taken place (Bass, 2005; CBMS, 2001, 2012; Ferrini-Mundy and Findell, 2001; McCallum, 2003; Wu, 2006). As a result, mathematics courses, which can be dramatically different from regular advanced courses, have been developed.

The inherent difficulties in working across institutions, faculties or departments can be a strong deterrent to joint actions. In some cases, mathematicians and mathematics teacher educators work in the same department, enabling collaborations and the emergence of educators who are experts in both domains. This creates opportunities, which are possibly reminiscent of the situation lived by earlier researchers in mathematics education. An example of research in this direction comes from Greece (Petropoulou et al., 2011; Karavi et al., 2020), where a strong expertise in mathematics and mathematics education enabled an individual to develop courses with a clearer expectation about students' difficulties, choosing representations enabling students to build a stronger understanding of advanced concepts. In this case, the MTE was both a mathematician and a mathematics education specialist.

However, it is a challenge in many cases to find how mathematics educators can support mathematicians in a study of their teaching and its impact on students' learning. As Bass points out, a fruitful collaboration with mathematics educators may not be practical for all mathematicians who wish to contribute to teacher education (Bass, 2005, p. 418). It also remains true that an important barrier is what some mathematicians expect from mathematics education research, such as the search for the effective teaching strategies, is very different from the views of mathematics education researchers (Sierpinska and Kilpatrick, 1998; Schoenfeld, 2000).

On a very positive side, in other settings, team-teaching between mathematicians and MTEs has taken place (Grassl and Mingus, 2007; Heaton and Lewis, 2011; Sultan and Artzt, 2005; Thompson et al., 2012). Research into the conditions enabling interdisciplinary collaboration in the way of team-teaching and joint work showed that shared goals, mutual trust, and open-mindedness (Goos and Bennison, 2018; Ponte et al. 2003) were key issues. It also showed that there was an initial fear of being judged by the other: mathematicians on their teaching, and mathematics educators on their mathematics.

Conditions hindering such work include cultural differences, grounded in epistemological differences between disciplines, as well as the lack of recognition, in both communities, of the value of such work (Goos and Bennison, 2018). There is also a difficulty for those working at the boundary between disciplines who can feel "like they belong to both one world and the other, or to neither one world nor the other" (Goos and Bennison, 2018, p. 272).

The emergence and the importance of brokers in such collaborative work is one of the important aspects considered in Goos and Bennison (2018): here, the collaborative work is not entirely in mathematics education or entirely in mathematics, and is done in such a way that actors from these distinct communities are engaged towards a common goal. When initiating collaborations, the presence and the emergence of brokers play a key role. They can be crucial in promoting and sustaining further work, and thus are both a product of successful collaborations and an ingredient for their ongoing success.

The presence of such brokers can also help in avoiding conflict or dealing more constructively when conflict arises. They play an important role in shaping the way one community views the other, and can help fight against ignorance and judgment, stereotypes and narrow views. This can be at the local, national and international levels, sometimes simply by sharing views about the complexity of the work done by "the other side".

As actors in different countries and institutions, many of us are in a situation where we can work towards enabling joint work by recognizing the importance of work done in collaboration in our respective communities. Despite the numerous challenges and obstacles, positive experiences show how much is to be gained from close collaborations.

2.2. Cultural aspects and their influence on the development of collaborations

Building joint actions among the actors involved in the education of prospective secondary school teachers is a long-term process and is framed by institutional, cultural, and political factors. What are some of the factors that have made this possible in different settings?

On the international level, and particularly at ICMI, several outstanding individuals have been recognized as genuine members of both communities. They were, generally, established research mathematicians who developed a very strong interest in mathematics education, in some cases leading to a career as a researcher in mathematics education: Hyman Bass and Michèle Artigue, two recent former presidents of ICMI, are prime examples of such individuals, and many other examples are found in Karp and Roberts' book (Karp and Roberts, 2014). These individuals have contributed to the development of the perspective that mathematics education is a genuine scientific endeavour, albeit very different from mathematics.

One important example motivated by ICMI has been the Capacity and Networking Project (CANP) which has conveyed the participation of mathematicians and mathematics educators in workshops held in different developing regions: Francophone Sub-Saharan African (Mali, 2011), Central America and the Caribbean (Costa Rica, 2012), Southeast Asia (Cambodia, 2013), East Africa (Tanzania, 2014), Andean Region and Paraguay (Perú, 2016).

In Norway, joint projects of mathematicians and mathematics educators have been established. The Erasmus+ European project PLATINUM (http://platinum.kubg.edu.ua/en/), which consists of seven European countries partnership between mathematicians and mathematics educators, aims to improve the teaching and learning of mathematics at the university level developing resources promoting inquiry-based learning. Mathematics education researchers and mathematicians contributed in different ways in the development of these resources and in their enactment.

In Canada, the establishment in 1978 of the Canadian Mathematics Education Study Group (CMESG) has led to annual meetings of mathematicians and mathematics educators in a highly collaborative work setting, around issues of mathematics education, particularly in mathematics teacher education. A Canadian community of mathematicians and mathematics educators was gradually formed, and numerous personal relationships forged. This enabled the development of mathematics courses for teachers influenced by mathematicians and mathematics educators (Hodgson, 2016). Initial teacher education has been an ongoing theme of working groups at these meetings, some examples are given in series proceedings of the annual meeting (Marynowski, Dufour and Liljedahl, 2017; Gourdeau and Nolan, 2016; Gourdeau, Oesterle and Stordy, 2014). The collaboration and joint involvement of mathematicians and mathematics educators in mathematics education discussion (through CMESG as well as in the Canadian Mathematics Education Fora of the Canadian Mathematical Society), and in the initial teacher education, may help explain why the Math Wars, which have affected the USA, have been much less intense in Canada.

3. Mathematicians, Mathematics Educators, Teachers and Other Actors in the Professional Development of Practicing Teachers

3.1. Joint actions in teacher professional development

Although in the initial teacher education there are research studies where mathematicians and mathematics educators collaborate, especially in designing and even team-teaching common courses (e.g. Bleiler, 2015), the research on collaboration between mathematicians, mathematics educators, and practicing mathematics teachers is rather rare. The discussion document of the ICMI-25 Study on "Teachers of mathematics working and learning in collaborative groups" (International Program Committee for ICMI-25 Study, 2019) addresses also as an important question the role of the different actors in teacher collaboration, including teachers, leaders, mathematicians, researchers in mathematics education. However, in the conference related to this Study that took place in Lisbon from the 3rd to 7th of February 2020 (http://icmistudy25.ie.ulisboa.pt/) there were no submissions reporting research in this area. Nevertheless, mathematicians and mathematics teacher educators are involved in supporting practicing mathematics secondary school teachers to develop their teaching. Mathematics educators usually organize practice-based professional development programs for teachers or act as facilitators in teacher collaborative groups (e.g. Cooper, Olsher and Yerushalmy, 2019). Mathematicians support teachers mainly by designing resources such as curriculum documents, textbooks, teacher's guides (e.g. Potari, Psycharis, Sakonidis and Zachariades, 2019).

Concerning the professional development of mathematics teachers, there is an increasing research interest on the collaboration between mathematics educators and mathematics teachers with a particular focus both on the process and the outcomes of collaboration (see the ICME international survey in (Robbuti et al., 2016)). In addition to the ICME international survey, we see several papers reporting collaborative efforts between mathematics teacher educators and mathematics teachers (Arbaugh, 2003; van Es, 2009) and a special issue in ZDM (issue 46) focusing on the collaboration addressing the importance of the joint actions, see (Jaworski and Huang, 2014). Collaboration between mathematicians/mathematics educators and teachers in the context of professional development has also been seen in offering professional development programs to teachers for revisiting advanced mathematics content that they had met during their university studies. An example of such a program concerns the teaching of linear algebra for ten practicing teachers in the US (Harel, 2017). Another example of collaboration among mathematicians, mathematics educators, prospective and practicing mathematics teachers is also reported in the study of McGraw, Lynch, Koc, Budak and Brown 2007) focusing of the use of multimedia cases as tools for teacher professional development. Through the analysis of online and face-to-face discussions, the authors show that the different backgrounds and experiences of the participants can blend in such a way that it promotes rich discussions about mathematics, teaching and learning.

In designing resources for teachers, we also see examples of collaboration between mathematicians, mathematics educators and teachers. In the study of Potari et al. (2019) that took place in Greece, different actors participated in the design of a new mathematics curriculum for the compulsory education. In that setting, tensions emerged between the different communities of participants while persons that participated in these communities (e.g. teachers who had also been involved in research in mathematics education), acted as boundary persons and facilitated the overcoming of the tensions. A similar example from China in improving teachers' teaching, university mathematics educators collaborated closely with mathematics teachers in designing and implementing lessons, and it was found that the identity of the participating teachers changed from "problem posers and solution receivers" to "collaborative problem solvers" in negotiating and finding solutions to practical problems with mathematics educators (Qi et al., 2021).

3.2. Mathematicians' efforts on teacher professional development with collaboration of other actors

The fact that there is no systematic research on the collaboration of mathematics educators and mathematicians in the context of professional development of practicing teachers does not necessarily mean that there are no such efforts in different countries. The members of the panel address collaborations that have taken place in their countries in the context of conferences and workshops, and annual conferences of the mathematics teacher associations.

More systematic collaborations are seen in China and the Russian Federation. In China, mathematicians have participated in many professional development programs for primary and middle school practicing teachers. Currently, one program is organized by the Ministry of Education (MOE) (named "Guopei" Project). It aims to improve teachers' professional skills, especially in rural areas. From the total of 45 mathematics expertise trainees in the first issued name list by MOE, 8 are mathematicians, which reveals the emphasis on the role of mathematicians from the government. In addition, mathematicians are involved in designing and planning mathematics curriculum standards and textbooks with other actors such as mathematics teacher educators and mathematics teachers. The two leaders of the current two mathematics curriculum standards (Ministry of Education [MOE], 2011, 2017) are all mathematicians. Among the six versions of current high school mathematics textbooks (Sujiao, Shanghai, Renjiao, Xiangjiao, Ejiao, BNU), two-thirds of chief editors are mathematicians who work in in-depth collaboration with other authors/actors (e.g., mathematics educators, teachers, Jiaoyanyuan, etc.).

In the Russian Federation, school textbooks and curriculum materials have been developed by working groups headed by leading mathematicians, while both mathematics educators and teachers participate in these groups. In this context, mathematicians are mainly responsible for the mathematics content while mathematics educators and teachers for the ways that the content can become accessible to the

students. Moreover, mathematicians often participate in the professional development of practicing teachers, and they pay much attention to the popularization of mathematics both among students and among mathematics teachers. Mathematicians design and develop online courses for raising mathematics content knowledge of practicing teachers. For example, on http://ptlab.mccme.ru/node/5107 teachers can find a course on combinatorics and probability. Another example is the summer schools for teachers (in particular, for teachers of mathematics), organized by the top Russian universities such as the Moscow State University and the Higher School of Economics, where working scientists and educators give lectures and workshops for in-service teachers.

Another example of joint action between mathematicians and mathematics educators, in which some teachers are involved, is given by Mathematics Olympiads for secondary education in Latin America. [For example, the Brazilian Math Olympiad for Public School Students (18 million yearly participants), a nationwide educational project, includes teacher training programs.] Another such collaboration has been around mathematics modelling in Latin America and China. There is also a rich and long-time tradition of joint action of mathematicians, future math teachers and math educators in the Mathematics Olympiads for school children in Russia.

One very particular experience was developed in Costa Rica, following an unusual political decision made by a minister of education. A group of mathematics education researchers from public universities (whose initial training was in mathematics) and some practicing teachers worked as a team (Mathematics Education Reform in Costa Rica Project, https://www.reformamatematica.net) to design a new mathematics curriculum for all Primary and Secondary education (approved in 2012).

The same team with the inclusion of technology specialists have participated in the implementation of the new curriculum with a special emphasis on virtual instruments: designing, developing blended courses (with face-to-face and online dimensions) for primary and secondary teachers and pedagogical advisors (2012–2017), avant-garde MOOC and Mini-MOOC courses (since 2014) for teachers, and high school students and many other innovative virtual resources since 2019 (Ruiz, 2018, 2020). Since 2012, this team has had the support of several ministers of public education during three different national governments (Ruiz, 2020).

Here two things can be emphasized: There was a cooperation between political actors, researchers, teachers, technology specialists, pedagogical advisors within a scenario of curricular design and development. And second, during the pandemic since 2020, when virtual educational strategies gained extraordinary relevance, the multiple materials produced by this team associated with the Ministry of Public Education have constituted a non-improvised base of pedagogical support for the student population.

4. Collaboration as a Community of Practice

In the previous sections, some effective examples of collaboration between mathematicians and mathematics educators have been reported. Some of these

examples share characteristics of a Community of Practice CoP (Wenger, 1998) in which mathematicians and mathematics educators are mutually engaged in an activity (e.g., co-designing, co-teaching courses), are held together by a joint enterprise (e.g., MOOC for teachers in the pandemic period), and have a shared repertoire of customs of practice (e.g., sharing views and experiences about the related resource). In this way, we can see the collaboration as forming a CoP. With consideration of seven principles of CoPs proposed by Wenger et al. (2002), we conclude three aspects:

- Achieving agreement on understanding the common objects
- Promoting equality in communication and mutual respect, and
- Facilitating transformation in identity.

To conduct the planned activities smoothly and efficiently in the collaboration, merely identifying common objects (e.g., co-designing course, co-developing curriculum, or textbooks) is usually not adequate given that mathematicians and mathematics educators/other actors may have different interpretations of them. Thus, one key factor is to ensure that every actor in the group has a common understanding of objects.

Based on the understanding of objects, promoting equality in communication and mutual respect is very crucial. A supportive atmosphere builds trust and enables mathematicians and other actors to express ideas and concerns openly (Henrick et al., 2017) without fear of others' judgment. In mathematicians' or mathematics educators' individual activity, most of them have only one fixed identity which guides their daily activities. However, during the collaboration, the objects require every member in the group to co-design the course for teachers, so that all members become co-learners and co-designers. Through their participation and collaboration, the mathematicians, mathematics educators, and other actors share and absorb each other's wisdom and sometimes act as "brokering" (Wenger, 1998) to facilitate transforming their old identities into the new ones of co-learners and co-designers. This type of benefit-sharing mechanism enables all the actors to work well with a clear understanding of the participants' identities in collaboration, performing their "delegations" from each community (mathematics content and education content respectively) and undertaking joint efforts for common development (Wenger, 1998).

Forming communities of practice in which mathematicians, mathematics educator researchers, teachers and other actors collaborate for contributing to initial teacher education and teachers' professional development is not an easy task. However, it seems that it is a promising way to offer prospective and practicing teachers learning opportunities that can have a positive impact on the mathematics education of students in schools.

5. Closing Remarks

Based on the aforementioned statements, we come to two main conclusions and propose four questions for further consideration.

Conclusion 1: *The relationships between the social agents involved in the preparation of teachers are not identical in various countries and regions due to diverse cultural or socio-economical contexts or individual or group experiences.*

For example, mathematicians may or may not participate directly in the design of curriculum materials for the school actors. Active teachers may or may not have a "say" in some teacher preparation programs.

Mathematicians can publish books on the history of mathematics or collaborate in publishing textbooks for pre-university education. In some countries there is no participation of mathematicians or even math educators in such activities.

Conflicts or tensions are not the same in all latitudes. The "Math Wars" in the USA were not a worldwide phenomenon, and in other countries the nature of conflicts may have been different.

So, it seems that it is not possible to offer a prescription for all settings.

However, we can enunciate

Conclusion 2: *It is always possible to identify internationally good practices that promote collaboration between educational agents and to manage conflict appropriately, but always with careful calibration of specific contexts.*

Even if conflict can be a problem for collaboration, they can also be an opportunity to calibrate the complexities of collaboration and to further develop these collaborations in fruitful ways. To conclude, we suggest some questions that could support research in this area as well as the emergence and development of such collaborations:

1. How to promote trust, mutual respect, and shared beliefs, values and goals, and stimulate joint practices among the several actors involved in mathematics teacher education? How can a community of practice can be developed and sustained?
2. What are the main features of institutional environments that facilitate collaborative work between the different actors?
3. What practices can help achieve convergence between the priorities and practices of universities and those of schools?
4. How to strengthen the participation of teachers in communities as a context for their professional learning? What is the role of the several actors in this process?

References

F. Arbaugh (2003). Study groups as a form of professional development for secondary mathematics teachers. *Journal of Mathematics Teacher Education*, 6(2), 139–163.

H. Bass (2005). Mathematics, mathematicians, and mathematics education. *Bulletin of the American Mathematical Society*, 42(4), 417–430.

S. K. Bleiler (2015). Increasing awareness of practice through interaction across communities: the lived experiences of a mathematician and mathematics teacher educator. *Journal of Mathematics Teacher Education*, 18, 231–252.

Conference Board of the Mathematical Sciences (2001). *The Mathematical Education of Teachers*. Washington, DC: American Mathematical Society.

Conference Board of the Mathematical Sciences (2012). *The Mathematical Education of Teachers* II. Providence, RI/Washington, DC: American Mathematical Society and Mathematical Association of America.

J. Cooper and R. Karsenty (2018). Can teachers and mathematicians communicate productively? The case of division with remainder. *Journal of Mathematics Teacher Education*, 21(3), 237–261.

J. Cooper, S. Olsher and M. Yerushalmy (2019). Didactic metadata informing teachers' selection of learning resources: boundary crossing in professional development. *Journal of Mathematics Teacher Education*, 1–22.

Educating the Educators II Conference, https://icse.eu/educating-the-educators-iii/

J. Ferrini-Mundy and B. Findell (2001). The mathematical education of prospective teachers of secondary school mathematics: Old assumptions, new challenges. In *CUPM discussion papers about mathematics and the mathematical sciences in 2010: What should students know*? Washington, DC: Mathematical Association of America, pp. 31–41.

M. Goos and A. Bennison (2018). Boundary crossing and brokering between disciplines in pre-service mathematics teacher education, *Math. Ed. Res. J.,* 30, 255–275. https://doi.org/10.1007/s13394-017-0232-4.

F. Gourdeau and K. Nolan (2016). Mathematics in teacher education: What, how… and why/Les mathématiques dans la formation des enseignants: quoi, comment… et pourquoi. In S. Oesterle, D. Allan, and J. Holm (eds.), *Proceedings of the 2016 Annual Meeting of the Canadian Mathematics Education Study Grou*p, Burnaby, BC: CMESG/GCEDM, pp. 137–150.

F. Gourdeau, S. Oesterle and M. Stordy (2014). Mathematical Habits of Mind / Modes de pensée mathématiques. In S. Oesterle and D. Allan (eds.), *Proceedings of the 2014 Annual Meeting of the Canadian Mathematics Education Study Group*, Burnaby, BC: CMESG/GCEDM, pp. 83–115.

R. Grassl and T. T. Y. Mingus (2007). Team teaching and cooperative groups in abstract algebra: Nurturing a new generation of confident mathematics teachers. *International Journal of Mathematical Education in Science and Technology*, 38(5), 581–597.

G. Harel (2017). The learning and teaching of linear algebra: Observations and generalizations. *Journal of Mathematical Behavior*, 46, 69–95.

R. M. Heaton and W. J. W. Lewis (2011). A mathematician–mathematics educator partnership to teach teachers. *Notices of the American Mathematical Society*, 58(3), 394–400.

E. C. Henrick, P. Cobb, W. R. Penuel, K. Jackson and T. Clark (2017). *Assessing Research-Practice Partnerships: Five Dimensions of Effectiveness*. New York, NY: William T. Grant Foundation.

B. R. Hodgson (2016). Apport des mathématiciens à la formation des enseignants du primaire: regards sur le «modéle Laval». In S. Oesterle, D. Allan, and J. Holm (eds.), *Proceedings of the 2016 Annual Meeting of the Canadian Mathematics Education Study Group*. Burnaby, BC: CMESG/GCEDM, pp. 3–44.

C. Hoyles (2014). Solid findings in mathematics education: the influence of the use of digital technology on the teaching and learning of mathematics in schools. *European Mathematical Society Newsletter*, 4–51

International Program Committee for ICMI Study 25 (2019). Teachers of mathematics working and learning in collaborative groups. Discussion document. Retrieved from https://www.mathunion.org/fileadmin/ICMI/ICMI%20studies/ICMI%20Study%2025/190218%20ICMI-25_To%20Distribute_190304_edit.pdf

B. Jaworski and R. Huang (2014). Teachers and didacticians: key stakeholders in the process of developing mathematics teaching. *ZDM Mathematics Education*, 46, 173–188.

T. Karavi, D. Potari and T. Zahariades (2020). Proof teaching at the university level: the case of a lecturer who is mathematician and mathematics educator. Paper presented at INDUM 2020 conference, 27–29 March 2020, Bizerte, Tunisia.

A. Karp and D. L. Roberts (2014). *Leaders in Mathematics Education: Experience and Vision*. Sense Publishers, Rotterdam.

K. Maass, P. Cobb, K. Krainer and D. Potari (2019). Different ways to implement innovative teaching approaches at scale. *Educational Studies in Mathematics*, 1–16.

R. Marynowski, S. Dufour, and P. Liljedahl (2017). Compréhension approfondie des mathématiques scolaires/Deep understanding of school mathematics. In J. Holm, S. Mathieu-Soucy and S. Oesterle (eds.), *Proceedings of the 2017 Annual Meeting of the Canadian Mathematics Education Study Group*, pp. 121–133. Burnaby, BC: CMESG/GCEDM.

W. McCallum (2003). Promoting work on education in mathematics departments. *Notices of the American Mathematical Society*, 50(9), 1093–1098.

R. McGraw, K. Lynch, Y. Koc, A. Budak, and C. A. Brown (2007). The multimedia case as a tool for professional development: an analysis of online and face-to-face interaction among mathematics pre-service teachers, in-service teachers, mathematicians, and mathematics teacher educators. *Journal of Mathematics Teacher Education*, 10(2), 95–121.

Ministry of Education, People's Republic of China (2011). The first issued list of expertise trainees for Guopei Project. http://old.moe.gov.cn/publicfiles/business/htmlfiles/moe/s4649/201103/116158.html.

Ministry of Education, People's Republic of China (2012). *Mathematics Curriculum Standard for Compulsory Education (2011 Version)*. Beijing: Beijing Normal University Press.

Ministry of Education, People's Republic of China (2017). *Mathematics Curriculum Standard of General High Schools (2017 Version)*. Beijing: People's Education Press.

G. Petropoulou, D. Potari, and T. Zachariades (2011). Inquiring mathematics teaching at the university level. In B. Ubuz (ed.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3. Ankara, Turkey: PME, pp. 386–392.

J. P. Ponte, I. Segurado, and H. Oliveira. (2003). A collaborative project using narratives: what happens when pupils work on mathematical investigations? In A. Peter-Koop, V. Santos-Wagner, C. Breen, and A. Begg (eds.), *Collaboration in teacher education: examples from the context of mathematics education*. Dordrecht: Kluwer, pp. 85–97.

D. Potari, G. Psycharis, C. Sakonidis, and T. Zachariades (2019). Collaborative design of a reform-oriented mathematics curriculum: contradictions and boundaries across teaching, research, and policy. *Educational Studies in Mathematics*, 102(3), 417–434.

C. Qi, C. Chen, and R. Huang (2021). Teacher learning through collaboration between teachers and researchers: A case study in China. *International Journal of Science and Mathematics Education*: https://doi.org/10.1007/s10763-021-10241-7.

A. Ralston (2004). Research mathematicians and mathematics education: A critique. *Notices of the American Mathematical Society*, 51(4), 403–411.