# Chapter 19
# New Math in Latin America (and a Glimpse at Costa Rica)


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**Abstract** The "modern mathematics" reform (New Math) in Latin America is described, with an emphasis on its main international agents on the continent: The Inter-American Committee of Mathematics Education (CIAEM) and the conferences that this organization nurtured. A distinction between the first four conferences that tried to propagate the reform and the fifth one is documented. In the latter, a separation from the New Math was evidenced and it began a new stage in the evolution of these agents. The particular experience of the reform in Costa Rica is included not only to provide details of a special case but to highlight characteristics that to some extent were also present in other countries of the region. The reform in Costa Rica will be contrasted with a new and very ambitious mathematical reform that the country launched in the second decade of the twenty-first century. It can be seen as a "tale of two reforms." The concluding remarks summarize some results that New Math (ideas and developments, or reactions toward them) provoked so far as the teaching of mathematics was concerned. Finally, comments on some elements of the current situation of CIAEM are offered.

**Keywords** Bourbaki · CIAEM · Costa Rica · Cuban revolution · Curriculum reforms · Howard Fehr · IACME · ICMI · IMU · Latin America · Luis Santaló · Marshall Stone · Mathematics education · New Math · Sputnik · Ubiratan D'Ambrosio · US School Mathematics Study Group

## Introduction

In the 1960s, New Math played an important role in Latin America in the development of the communities associated with mathematics and its teaching. It was an *external* factor to this region strongly determined by the leading role of distinguished mathematicians from the USA, with the support of mathematicians associated with the intellectual perspectives formulated by the Bourbaki group (born in the mid-1930s in France). What were the individual and collective agents and ideas that developed this reform in the region? Its main moments? How did these agents and ideas evolve over the years to the present day? Before beginning to answer these questions, we will briefly delve into some of the conditions that defined the historical setting of the New Math and that had a connection to the development of the reform in Latin America.

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## Three Factors Within the Modern Mathematics Reform

To begin, it should be recalled that at the 1958 International Congress of Mathematicians in Edinburgh, five participants from the USA gave a key report that proposed the need for a reform in the teaching of mathematics associated with the New Math: Edward G. Begle, Howard F. Fehr, Robert E. K. Rourke, Marshall H. Stone, and Albert W. Tucker (Fehr et al. 1971). Begle, Fehr, and Stone would later be involved with reform in Latin America (Barrantes and Ruiz 1998).

Although there were many elements in the ideas proclaimed by supporters of the New Math, one, in particular, was especially meaningful for Latin America—that was the "war cry" expressed by Jean Dieudonné at the Royaumont Seminar in 1959: "Euclid must go!" (another version: "Down with Euclid!"). At that time any attempt to eradicate Euclidean geometry from Latin American schools would have been a very "touchy" issue for teachers and scholars of the region.

**The Social Factor: The Call to Mathematicians** In the late 1950s there was a clear gap between university and pre-university mathematics in Costa Rica, and also in many other nations. There was a need to modernize the teaching of mathematics. In the broadest perspective, modernization would start from the necessity to adapt mathematics training to the scientific and technological development of the main western societies, although other factors were also present.

In this context, the idea emerged among mathematicians that they had the *historical mission* of getting involved in the pre-university teaching of mathematics and, in addition, defining what the modernization of mathematics should be and the establishment of the most appropriate bridges to university mathematics. Mathematicians of the highest level and prestige took on a heightened role that would have an impact on their communities of teachers and academics in their worlds (Moon 1986), and this happened with a particular force in weaker scientific and educational communities.

**The Intellectual Factor: Bourbaki** Within the mathematical community, those of the Bourbaki group had an especially important role, and that was especially true of those who embraced what we can call the "Bourbaki ideology" (see Chap. 3 in this volume). One of the aims was to "reorganize" or "reconstruct" mathematics with special participation of the notions of set theory, relations, and functions. However, the fundamental notion was that of *structure* (Cartier 2010).

In this ideology, there were theoretical assumptions, explicit or implicit—for example, that mathematics constitutes a single body, with a language and a conceptual logic that could account for all its parts. According to Barrantes and Ruiz (1998), the Bourbaki "ideology" was fed by dominant ideas in Western philosophy, such as mathematics is *a priori* knowledge apart from experience, and, therefore, its "truths" are absolute and infallible. Something close to a *paradigm* (in the sense of Thomas Kuhn) was adopted, and this was supported by a very broad and heterogeneous intellectual community with mathematicians in the foreground.

After World War II a relationship developed between US mathematicians and French Bourbakists. The Bourbaki manifesto *L'Architecture des Mathématiques* (1948) was translated into English and published in 1950 in the *American Mathematical Monthly* (Volume 76, pp. 221–232). Bourbaki's books were reviewed (in positive terms) in the *Bulletin of the American Mathematical Society* from 1953. Dieudonné himself taught at the University of Michigan during 1952, and then from 1953 to 1959 at Northwestern University. Weil taught at the University of Chicago from 1947 to 1958 and later at the Institute for Advanced Study, Princeton. Stone was Director of the Department of Mathematics of the University of Chicago between 1946 and 1952 and continued as a professor at that institution until 1968. Stone hired Weil. These professional contacts between mathematicians on both sides of the Atlantic were important for explaining the convergence of ideas and projects that emerged in and helped define the New Math, and this came to be manifested in Latin America

**The Political Factor: The Cold War** Another influencing factor was the launch of Sputnik on October 4, 1957. That "frightened" the USA within a Cold War scenario. Just a couple of years later, in a region that the North Americans considered their "backyard," the Cuban Revolution took place. This generated an atmosphere of alarm that would have an impact on the mathematics reform, generating broad institutional support and the injection of money which would underpin the New Math.

In the world, these three factors—social, intellectual, and political—were intermingled with different weights and roles in various countries. Historians need to consider whether mathematicians leveraged the political factor to position themselves advantageously against other academics (e.g., physicists). Alternatively, did the politicians or institutions use mathematicians and scientists to advance their plans and ideological objectives? To what extent were the basic ideas of the reform the central motivation for action on the part of mathematicians? Some of those questions are not easy to answer, of course, with general reasons and causative factors difficult to identify and untangle.

## Reform in Latin America

In Latin America, the ideas and agents of the New Math were imported—mainly from the USA, but also from Europe.

A first US import was the textbooks of the School Mathematics Study Group (SMSG) which had been prepared under the leadership of Edward Begle. However, they were not the main agents introducing New Math to Latin America. Barrantes and Ruiz (1998) suggested that the decisive initiating factor was the First Inter-American Conference on Mathematical Education (I CIAEM or IACME I) and the creation of the Inter-American Committee on Mathematics Education in 1961. [Here CIAEM will be used most of the time as an acronym for *Comité Interamericano de Educación Matemática* (Spanish), *Comitê Interamericano de Educação Matemática* (Portuguese) or Inter-American Committee of Mathematics Education, or the name of the conferences associated with this Committee.]

In 1961, two years after Royaumont, the I CIAEM was held in Latin America. One might wonder why this occurred so early, in this part of the planet? The most profound answer is likely to rest on rather personal reasons: The initiative came from Marshall Stone, a central figure for this reform in the world. Stone's dynamism, and particularly his special vocation for the Latin American region (something that accompanied him for many years), need to be taken into account. In addition, the influence, and participation in the region, of French mathematicians helped pave the way. But here, too, something similar to the Sputnik factor intervened—specifically, the "Fidel factor." The 1959 Cuban Revolution weighed particularly on the political class in this region. The Organization for American States (OAS) and other international organizations joined the effort to "neutralize the advance of communism" in the Americas, and this promoted educational and scientific actions. One of the programs that worked toward that end was the "Alliance for Progress," a plan in which Colombia had an important role (Arboleda 2019). One might think that Bogotá was not chosen by chance for this first CIAEM.

A second CIAEM conference was held in Lima in 1966 to follow up and reinvigorate the reform. This second conference came quite a few years after the first one to be held. One reason for that was that the reformers did not find sufficient financial and institutional support after 1961 to go ahead. In this second CIAEM, Stone indicated some of the difficulties in relation to the committee that had emerged from I CIAEM:

> The committee then had to find some way or securing a degree of permanence and of laying out a suitable mode or operating. At first the committee floated in the air with no official point of attachment and very little financial support. We explored various ways of transforming ourselves into a somewhat more permanent international body. This was a rather difficult thing to do. (Stone 1966, p. 20)

However, reform tactics were developed, as we will show later. In fact, it was relatively easy for the reform to penetrate Latin America. The progress of the reform was affected by the following:

- The role of universities, which generally became involved in the process in different ways, at different times, and at different levels of intensity.
- The participation of graduate students in mathematics returning from the USA and Europe, who supported—in general—the reform.
- The participation of scholars from the USA and Europe who were involved within the mathematical communities and their teaching in the region; some even stayed to live in these latitudes.
- The impact of textbooks, whether as translations of texts (from the USA or Europe) or those produced in the region.

These elements were interwoven. Universities invited or hired mathematicians; in-service and initial teacher preparation programs were created; university scholars or students translated or wrote texts, organized or participated in events (meetings and conferences, for example), and participated in or designed projects.

The Inter-American Committee of Mathematics Education should be considered the "agent" of the reform. Stone was appointed as its president in 1961 and continued in that position until 1972. Luis A. Santaló (born in Spain and based in Argentina) began in 1966 to represent Stone in Latin America (Fehr 1962a, 1966a). Stone's role for more than a decade was decisive. He became the central promoter of CIAEM and the conferences, the point of reference for the New Math in the Americas. One might say it was a true "Stone Age." However, Stone was not the only person to play a crucial role. For example, Professor Howard F. Fehr was systematically active in all the first conferences, as a central organizer and editor of the works emanating from them. He participated in the I, II, III, and IV CIAEM. The importance of his role has sometimes not been sufficiently recognized.

The influence in Latin America of the USA and French mathematicians did not come only from the CIAEM. Cartier (2010), for example, stated that in the case of Bourbaki in Latin America this influence occurred because Alexander Grothendieck, Dieudonné, and Weil spent many years in Brazil. Indeed, Dieudonné had been a professor at the University of Sao Paulo between 1946 and 1948, Weil was there from 1945 to 1949, and Grothendieck from 1953 to 1955.

Another element to be noted that played a role in the development of the reform was the creation in Argentina in 1959 by UNESCO of the Regional Center for Mathematics for Latin America. This led to the participation of young professionals from various parts of Latin America (Tirao 2018) who would later spread the reform in the region. One can recall the visits to Argentina by Bourbakists such as Laurent Schwartz between July and September 1958, and Charles Ehresmann in 1959 (Fernández-Stacco 2011), but probably the one that had the most significant impact on the support of New Math ideas in the region was Dieudonné's visit in the early 1960s. Tirao (2018) points out that "under the auspices of UNESCO and the Faculty of Exact and Natural Sciences of the University of Buenos Aires, Professor Jean Dieudonné of the University of Paris dictated—during the months of July to September 1962—a course in Buenos Aires" (p. 67). In this course Santaló and César Carranza participated. Carranza from Peru, was a student at that time but later played a prominent role in the CIAEM and was a member of its executive committee between 1975 and 1979.

The special relationship of Argentinian and Brazilian intellectuals with Europe made it easier for Bourbaki ideas to become known, and accepted, within Latin America.

## *Stone and the Inter-American Perspective*

The role of Marshall Stone in this reform in Latin America cannot be fully understood without reference to the first visit he made to this region and his work with mathematicians from this region over many years. Parshall (2007) characterized Stone as a "mathematical 'good neighbor'." Stone

made a very important visit to South America in 1943, at a time when he was president of the *American Mathematical Society*. According to Parshall (2007), George David Birkhoff (who had been Stone's thesis advisor at Harvard University) visited Argentina in 1942 in the spirit of "policy of the good neighbor" promoted by Franklin Delano Roosevelt in the 1930s (particularly on technological and scientific issues), and upon his return, he influenced Stone to continue in that direction. Stone was based in Buenos Aires between July and October, from where he took the opportunity to visit other cities in the region. It is interesting that in Lima he delivered a lecture in Spanish. Stone returned to the USA with the intention of supporting the participation of Latin American students in good American universities. Parshall (2007) commented:

> As Stone's efforts in the late 1940s and early 1950s to find an International Mathematical Union suggest, however, mathematics by the 1950s had become a more truly international, worldwide endeavor. It was bigger than one country or group of countries. It is increasingly dependent on worldwide collaborations and cooperation, not just targeted initiatives like that sparked by Roosevelt's "good neighbor" policy. (p. 29)

Apart from political considerations that intervened in the development of the New Math, it is clear that Stone also had "good neighbor" purposes that effectively in the early 1960s took on much different broader international and organizational perspectives. These individual "drives" cannot be dismissed in the history of the New Math in Latin America. One might be led to think, for example, that this reform in the region was conceived only as a political move calculated to prevent the development of communism, but the origins of the "Inter-American" perspective that is coined in the very names of the committee and conferences seem to be here.

## *The Reform Through the CIAEM and Its Conferences*

**Bogotá, Colombia, 1961: Definitions** The I CIAEM was held in Bogotá, Colombia, from December 4 to 9, 1961 (Figure 19.1). It was sponsored by the International Commission on Mathematical Instruction (ICMI) and, significantly, by the Organization of American States (OAS). It also received important support from the National Science Foundation of the USA. Mathematicians and mathematics teachers, representatives, or guests from 23 American countries participated. The Colombian Minister of Education, Jaime Posada-Díaz, officially opened this event. It should be noted that Posada-Díaz was part of the government of Colombian President Alberto Lleras-Camargo who, as Arboleda (2019) indicated, was the first secretary of the OAS and a crucial figure influencing US politics at the time.

The main intellectual orientations of the conference were as follows:

- To change the teaching of geometry in secondary education, by adopting the point of view of linear algebra rather than the teaching of Euclidean geometry.
- To teach mathematics through the study of basic structures (in order to highlight its unity), for which the teaching of modern algebra was very important.

Fehr (1962b) was very "strong," in supporting Dieudonné at Royaumont: "Euclid's geometry [...] is, today, sterile, it lies outside the main path of mathematical advances and can be relegated without fear to the archives for the use of the historians of tomorrow" (p. 38). He added that in secondary education the essentials of Euclidean geometry can be given in two or three months freeing time to work with algebra, studying new number systems and algebraic structures, and, finally, combining algebra with geometry studying affine plane geometry. The aim should be to lead students quickly to the study of vector spaces. That said, Fehr, himself indicated that axiomatics should not be given too much emphasis at this level (Barrantes and Ruiz 1998).

Gustave Choquet (who never fully embraced Bourbakist ideas but nevertheless identified with their "ideology") insisted that the special Bourbakist vision of modern mathematics should influence secondary education. Among the ideas that he presented were that teaching at all levels should be reviewed based on the discovery of the great structures and that efforts should also be made to unify teaching at all levels. He stated that "We will see an ever-increasing unity in mathematics and greater unity in the teaching of the subject at all levels. The slogan will be algebra and fundamental structures from kindergarten to the university!" (Choquet 1962, p. 76). Choquet was very clear: All teaching based on the historical method was inconceivable. Although he proposed that students should quickly be put in contact with unifying concepts and basic structures he was aware that there was a danger of using axiomatics prematurely.

For Stone (1962), it was possible to teach modern algebra in high school to the point where it was possible to include rings of polynomials over a field. Begle (1962) explained the way in which reform in mathematics education was being carried out in the USA. He summarized the role of the SMSG as a search for a program for schools that provided materials and guides for the preparation and training of teachers and added that SMSG was strongly supported by the National Science Foundation. Stone and Fehr have often been recognized as prominent reform figures, but it cannot be overlooked that Begle played a central role in US reform by influencing teachers and powerful related organizations (Vogeli 1976).

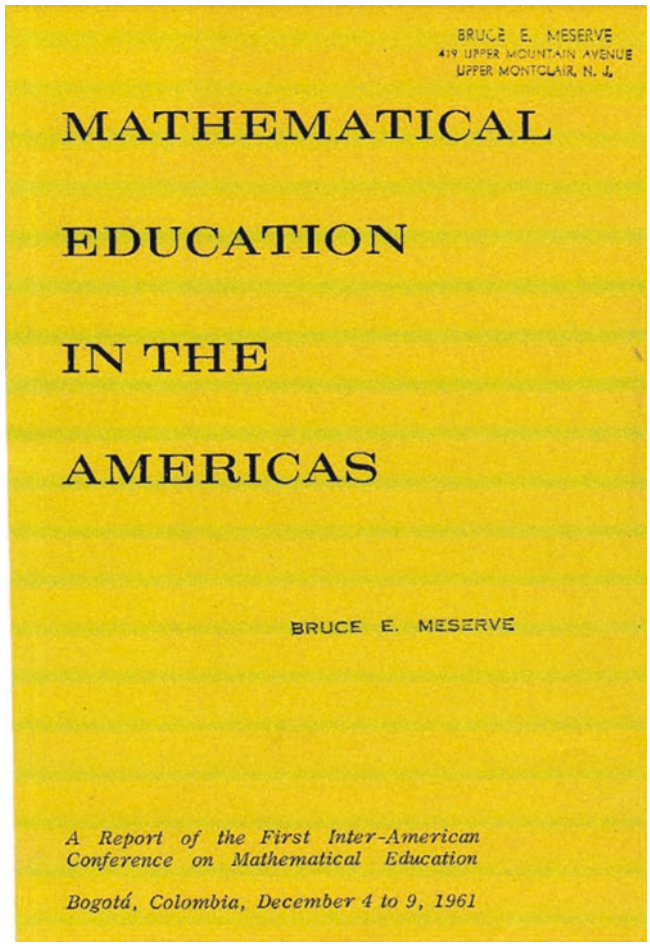


*Figure 19.1* Cover of I CIAEM Proceedings

The presentation by Schwartz (of France) was concerned with "The role of mathematics in physics, from the point of view of science education" (Schwartz 1962).

From this first conference, however, there were proposals that showed the difficulties that existed to implement this reform in the region. For example, the presentations by Andres Valeiras (Argentina), Santaló (Argentina-Spain), and Omar Catunda (Brazil) alluded to a situation in the teaching of mathematics in Latin American countries that included a shortage of university-prepared teachers, poor training, counseling difficulties, etc. It should be noted, too, that some other participants did not agree with the approach of Choquet, Fehr, and Stone (Arboleda 2019), and, probably, their criticisms were not well-publicized because of the prominence of the main speakers and central organizers of the event. Catunda (1962) however, was very clear: "what I would proclaim for Brazil would not be 'Down with Euclid!', but 'At least Euclid!'" (p. 55). Geometry was a sensitive symbolic topic. This "subterranean" critique of the New Math would gradually be expanded in conferences that would follow.

A problem was to devise ways of giving continuity to the results of the conference and the reform. It was agreed to *suggest* to the International Mathematical Union (IMU) "The creation of an Inter-American Commission on Mathematics Education, of a permanent character, for the purpose of providing continuity to the projects and ideas discussed in this Conference and to promote action calculated to raise the level and efficiency of secondary school and university teaching of mathematics" (Fehr 1962a, b, p. 168). Until this commission was arranged, a *pro tempore* committee was appointed consisting of Marshall Stone (USA), president, Alberto González (Argentina), Bernardo Alfaro-Sagot (Costa Rica), Alfredo Pereira (Brazil), and José Tola (Peru) (Fehr 1962a, b).

It is important to underline that three of the five US mathematicians who championed the New Math reform at the Edinburgh Congress in 1958 were in Bogotá. This was evidence of the relevance given by these reformers to this conference and of the convening power and influence of Stone. Similarly, the presence of French mathematicians of the highest level associated with the Bourbaki group or its ideology should be noted.

**Lima, Perú, 1966: An Assessment** The II CIAEM (Second Inter-American Conference on Mathematical Education) was held in Lima, Peru, from December 4 to 12, 1966 (Figure 19.2). The Minister of Education of Peru, Carlos Cueto-Fernandini, inaugurated the conference.

Stone (1966) in his opening address set out the agenda as follows:

> In the first place, it is natural that we wish to review what has been taken place in the hemisphere since the first Inter-American Conference on Mathematical Education, held almost exactly five years ago, in Bogota, Colombia. We must now ask: What had the report of that conference to do with what has been taken place in the last five years? Have its recommendations had any influence at all? Have some of them proved to be less practical than we had supposed at the time when we formulated them? In which countries has progress been most marked? (p. 11)

This II CIAEM focused on three topics:

- Assessment of what has been done in the time elapsed between the first and second conference.
- Problems related to mathematical preparation in the transition from high school to university.
- The preparation of teachers who would teach mathematics at the primary and secondary levels (Fehr 1966a).

The ideas of New Math were treated very seriously. Fehr (1966b) insisted:

> To achieve our objective, it is not enough to get rid of obsolete subjects, nor to replace them by subjects of a more modern variety, nor to graft a few modern concepts onto an outdated program. *School Mathematics must be reconstructed by making use of mathematical structures.* (p. 136) [italics by the author]

In the second part of the Conference, 22 delegations presented their reports: Argentina, Bolivia, Brazil, Canada, Colombia, Costa Rica, Chile, Ecuador, El Salvador, the USA, Haiti, Jamaica, Mexico,

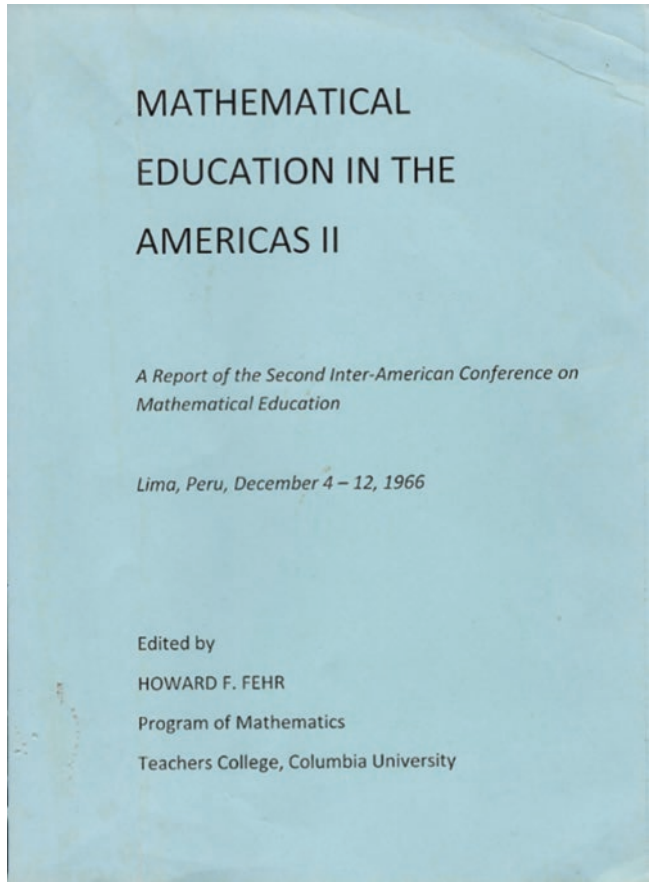


*Figure 19.2* Cover of II CIAEM Proceedings

Nicaragua, Panama, Paraguay, Peru, Puerto Rico, Dominican Republic, Trinidad and Tobago, Uruguay, and Venezuela. However, the reports were "informal" in the sense that they were not official from the governments but were simply the assessments of participants from each country.

Barrantes and Ruiz (1998) affirmed that unequal progress among countries in the Americas was found. For example, in Argentina, Brazil, Canada, and the USA, the reform process had generated profound changes in the contents of mathematics programs in secondary and higher education, and also through new approaches to teacher education and training. In Costa Rica, Chile, and others, new mathematics programs were enacted for the secondary level. In Ecuador, these programs began to be used only in some institutions as a pilot plan. In Bolivia, it had not been possible to carry out any kind of changes. In general, it was reported that partial or total changes had been made in the secondary school study programs. In some cases, there were also adjustments in the teacher preparation programs, and in many cases, training sessions were held for teachers. Most of the delegates referred to problems that had arisen in their own countries: Difficulties manifested themselves in the initial or in-service training of teachers, and additional human, economic, and operational resources were needed to carry out reforms successfully. Perhaps the main problem was that in many countries, teacher preparation was poor.

Several countries expressed their need to produce textbooks, consistent with the guidelines of the New Math and the first conference.

During the period from 1961 to 1966, the *pro tempore* committee became *the committee.* IMU had not created the "commission" that had been agreed to in Bogotá. And, in fact, since then there had been no further mention of a possible "commission," only the Inter-American Committee of Mathematics Education. This "committee" was not conceived as an organization or a movement, but as a relatively small group of people with some representation from the Americas. Also, it was viewed as a reference point for the teaching of mathematics in the region.

One of the issues that is clear from the proceedings of these conferences is the relationship of CIAEM (or IACME) with IMU and ICMI. In Lima, in the closing session, on December 12, 1966, the following was agreed: "The Inter-American Committee on Mathematics Education (IACME) originating in the First Inter-American Conference on Mathematical Education, December 4–9, 1961, is a non-governmental entity, affiliated to the International Union of Mathematicians, through the International Commission on Mathematical Instruction" (Fehr 1966b, p. 445). The CIAEM was conceived here as a technical body strongly associated with the conferences and with duties between the realization of these. It was at the Lima conference that the Committee was recognized or officialized, although not as a commission of IMU, only as "an autonomous regional body affiliated to ICMI" (Stone 1966, p. 20).

The second conference appointed the following Committee: Marshall H. Stone (USA) as president, César Abuauad (Chile), Ricardo Losada (Colombia), Manuel Meda (Mexico), Leopoldo Nachbin (Brazil), Luis A. Santaló (Argentina), Juan Jorge Schaffer (Uruguay), Edgardo Sevilla (Honduras), and José Tola (Peru). The Committee, later, appointed Santaló as vice-president, and Schaefer as secretary.

Barrantes and Ruiz (1998) commented that in Lima the CIAEM worked from a very ambitious agenda. The idea continued to be to advance New Math by introducing it to middle school and teacher centers. But the recommendations at this conference were more precise. In Bogotá, general ideas about modern mathematics and the need for all countries to become involved in it were presented. In Lima, the emphasis was on how certain processes were being carried out in the places where more progress had been made, especially with regard to teacher initial and in-service preparation.

1961–1966 was the most significant period for the New Math in the Americas. As CIAEM and its conferences evolved, the place given to New Math evolved too.

## The Road to Breaking with New Math

### *Three Conferences*

**III CIAEM: Bahía Blanca, Argentina, 1972** The objectives of the third conference were not as clear as those of the first two conferences. Many of the organizations and institutions previously involved seemed to have lost interest. In his opening address, Stone complained about the difficulties that had arisen in carrying out the reform and drew attention to an apparent lack of interest of many organizations (Barrantes and Ruiz 1998; CIAEM-UNESCO 1973). A different political atmosphere now seemed to prevail.

From this III CIAEM, more specific topics were selected on which talks and discussions would revolve. However, although the reform's aim was continued, it was clear that the reform had not turned out as planned and that serious difficulties had arisen. As Lore Rasmussen (1973), a teacher from the USA, stated:

> In practice, the reform fell short of many of its goals. Concerns for precise language such as the distinctions between number and numeral and equivalent and equal were artificially imposed. The insistence on implanting

> the language of sets, on the abuse of certain notations, the mention of commutative, associative and distributive laws, many times hindered the intuitive confidence for mathematics both in teachers and in students. (p. 95)

Rasmussen was critical of the way the reform had been carried out and proposed the need to consider psychological aspects in greater detail.

**IV CIAEM: Caracas, Venezuela, 1975** Ubiratan D'Ambrosio (Brazil) took on a very important leadership role at the fourth conference. He quickly wielded a perspective that emphasized the realities of mathematics education in Latin American countries and proposed that the teaching and learning of mathematics should relate to the progress of these countries and in particular to the inhabitants of the regions involved. His proposal criticized any focus of curricular dimensions on abstract mathematics and on cultural traditions different from those in Latin American regions (D'Ambrosio 1975). This period coincides with D'Ambrosio's intellectual construction of ethnomathematics, and his approach showed that there were already a vision and purposes different from the objectives assumed in the previous period.

It is of historical interest to record the speakers of two panels that expressed a "mixed" composition of the invited experts. According to CIAEM-UNESCO (1975):

- Mathematics and development. Daniel Crespín (Venezuela), Ubiratan D'Ambrosio (Brazil), Paul Dedecker (Belgium), Carlos Ímaz (Mexico), and Hernando Mateus (Colombia); moderator: Jesús Andonegui (Venezuela).
- The problem of the reform of mathematics teaching. Emma Castelnuovo (Italy), Luis Roberto Dante (Brazil), Jean Dieudonné (France), Howard F. Fehr (USA), Ricardo Losada (Colombia), Artibano Micali (France), Saulo Rada (Venezuela), and Willy Servais (Belgium); moderator: Tania Calderón (Venezuela).

Dieudonné and Fehr occupied an important place in the second panel. First, the participation of D'Ambrosio and Ímaz should be noted. The latter was a key figure in mathematics education in Mexico.

**V CIAEM: Campinas, Brazil, 1979** In this conference, due to its topics, speakers, and organization, there was a clear difference from the past. Criticisms of previous ideas were expressed openly. For example, in connection to geometry:

- Luis A. Santaló (Argentina-Spain), who had been president of the Committee since the IV CIAEM, pointed out that the panel addressing "Causes and effects of current trends in the teaching of geometry" had not accepted the idea of presenting geometry to young students from a purely axiomatic point of view. He maintained that the problem arose from confusing mathematics as a research discipline and mathematics as a formative and informative discipline (Barrantes and Ruiz 1998).
- Emilio Lluis (Mexico) also expressed the difficulties presented by the attempt to substitute the teaching of Euclidean geometry in the usual way to present it from the point of view of linear algebra and the loss that this represented in terms of pedagogy (CIAEM 1979). Lluis was elected one of the vice-presidents of the CIAEM for the period 1979–1985.

## *An Assessment of the Five Conferences*

We begin with the conferences' structures. The third and fourth conferences maintained a similar structure to the first two: Four topics considered important for mathematics education in the American countries were on the agendas, and lectures, communications, and panels were linked with those topics. Presentations of reports on the state of mathematics teaching in the countries were important as

they offered data and opinions which would make it easier to make decisions and recommendations. Furthermore, the participants were part of a community essentially "united by the reform."

The V CIAEM was drastically different. On this occasion, there were three plenary conferences with free topics. Then four central topics were discussed in detail under the modality of panels. The idea was to encourage a greater participation of the participants. Additionally, some workshop seminars with very specific topics were developed, and there was a large number of oral communications by the participants who put forward specific proposals, experiences, and ideas on different aspects of mathematics and its teaching. Although there were reports from country participants, they did not seem to be as relevant as the reports in previous conferences.

In the first four conferences, the primary aim was to change the type of mathematics taught at the secondary or primary levels in the participating countries. The accent was on curriculum reform: Design, syllabuses, programs, resources, and mechanisms for teacher preparation were considered. There was little consideration given to psychological and pedagogical aspects.

The first two conferences had a "missionary" character, but the others lost that perspective. The third one represented the beginning of a transition, and in the fourth, theoretical perspectives different (or contrary) to the New Math were raised. In the fifth conference, there was clearly a substantial change. There was a greater concern for more specific issues, especially of a pedagogical nature.

The first four conferences dealt with issues that would have an extraordinary impact on the nations of Latin America. But at the fifth conference, the political contexts had changed, and so had mathematics education across the world.

According to CIAEM-UNESCO (1973), Stone declined the presidency of the Committee for the III CIAEM. Santaló was elected president and held that position until 1979. It should be noted that D'Ambrosio was first elected vice-president at the IV CIAEM and then president from 1979 to 1985 (Figure 19.3). The composition of the CIAEM also changed significantly from 1979.

Two circumstances were important for the evolution of the CIAEM: The celebration of the Third International Congress on Mathematical Education (ICME-3) in 1976 in Karlsruhe, Germany, and the V CIAEM in 1979. ICME-3 at Karlsruhe had a more open structure than the previous ICMEs with the incorporation of professionals from the developing world. In particular, D'Ambrosio was part of the

*Figure 19.3* VI CIAEM, Guadalajara, Mexico, 1985, from left to right: E. Sebastiani (Brazil), A. Ruiz (Costa Rica), U. D'Ambrosio (Brazil), I. Harding (Chile), E. Lluis (Mexico), and G. Sánchez Vásquez (Spain)

International Program Committee and was in charge of a survey report "Objectives and goals of mathematics education. Why teach mathematics?" (D'Ambrosio 2008). The V CIAEM in Campinas, organized by D'Ambrosio, also represented a crucial change. There was a move away from the influence of the New Math both in the structure of the conference and in the topics considered. An influence of ICME-3 on the events and ethos of V CIAEM was evident (Ruiz 2013b).

## *IACME and the Support from ICMI, IMU, and Other International Institutions*

IACME's relationship with ICMI and IMU has had historical ups and downs; some associated with the region and others with the more general international situation experienced by these latter organizations, and also influenced by individuals. With Marshall Stone, the relationship was very close as he was the president of ICMI (1959–1962) when he founded IACME. Stone, being such a distinguished mathematician, was able to arrange for the participation of mathematicians from the USA and Europe in conferences in Latin America. ICMI authorities were at the conferences. For example, Hans-Georg Steiner (Germany), who was vice president of ICMI 1975–1978, and André Revuz (France), who was a member of the ICMI executive committee in 1967–1970, were in II CIAEM.

D'Ambrosio (2008) pointed out that prior to V CIAEM there was even greater influence from the *Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques*[1] [International Commission for the Study and Improvement of Mathematics Teaching] than from ICMI. However, at III CIAEM in Bahía Blanca in 1972, Hans Freudenthal was a keynote speaker. Freudenthal had been president of ICMI between 1967 and1970 and previously he had been on the executive committee from 1963 to1966. With D'Ambrosio, however, there was a renewal of the relationship between ICMI and CIAEM. In fact, D'Ambrosio was vice president of ICMI between 1979 and 1982, being at the same time president of IACME.

In the following years, several IACME executive committee members were on the ICMI executive committee. Between 1987 and 1990, Emilio Lluis was vice president. And, years later, two presidents of IACME were members-at-large of the executive committee—Eduardo Luna (1991–1994) and Carlos Vasco (1995–1998). More recently, IACME's relationship with ICMI and IMU has been even closer: The president of IACME (Angel Ruiz) was a member of the International Program Committee of ICME-11 (2008), vice president of ICMI between 2010 and 2016 (two terms), and also a member of the IMU Commission for Development Countries from 2011 to 2018. As Sánchez-Fernández (2019) stated: "These positions offered very valuable opportunities to strengthen IACME ties with ICMI and IMU and, in particular, promote actions in Latin America" (p. 174).

**International Support for IACME** Finally, it is important to mention that IACME had relevant financial support from US institutions only for the first conference. Although Stone had hoped that ICMI-IMU would support the IACME committee as it emerged from Bogota, that did not happen. Stone acknowledged the importance of the attachment to ICMI-IMU, and he was not happy with the lack of support; especially financial support. That was something he made clear at the II CIAEM:

> If anyone asks what the official status and the authority of the Committee are, the answer now is as follows: The conference at Bogota was called and organized by the International Commission on Mathematical Instruction in the International Mathematical Union; that conference elected the Inter-American Committee on Mathematical Education (IACME) with authority to organize and modify itself and the duty of implementing so far as possible the recommendations of the conference; and IACME is now an autonomous regional body affiliated to ICMI in accordance with the latter's statutes and terms of reference.

[1] See Chap. 3 in this volume.

> That then is where we stand today. We do have an official point of attachment or affiliation, and we have practically no money. (Stone 1966, pp. 20–21)

And in III CIAEM he insisted:

> It has been very difficult to organize this congress and impossible to put into practice the first outlined program. The reason, in a word, is money. The scarcity of funds reflected that the type of cooperation represented by the Inter-American Committee on Mathematical Education was no longer interesting. Mathematical societies, teachers' associations, ministries of education, other national agencies, foundations and international organizations such as the International Commission on Mathematical Instruction, OAS and UNESCO have not shown interest in supporting the work of IACME as they should. (Stone 1973, p. 18)

Stone wanted all the countries that were represented within the IACME to contribute yearly to support a more permanent entity. That never happened. He was very disappointed with the lack of funding and support (especially from the US institutions), and never participated again in an IACME conference. US institutions never again supported IACME after the II CIAEM. A "change of epoch" occurred, probably connected to the more general political atmosphere. However, IACME began receiving stronger support from UNESCO with the third conference (firstly from its office in Montevideo, and then from the mathematics education office in Paris, which was run by Edward Carl Jacobsen during the period from 1976 to1992).

## *Details on Names*

The first two conferences were named "Inter-American Conferences on Mathematical Education," but at some point during the following conferences, they became "Inter-American Conferences of Mathematics Education."

In I CIAEM the resolution on the organization to be suggested to IMU was "Inter-American Commission on Mathematics Education" and there was elected a *pro tempore* committee. In the II CIAEM in relation to the committee, the terms used were "Inter-American Committee on Mathematical Education" (Fehr 1962a).

The conference proceedings published by UNESCO (1973, 1975, 1979, 1990, 1992) stated: "*Comité Interamericano para la Enseñanza de la Matemática*" [Inter-American Committee for the Teaching of Mathematics]. However, *within* the proceedings of the III and IV CIAEM, it was "*Comité Interamericano* de *Educación Matemática*" [Inter-American Committee of Mathematics Education]. And the conferences were: "*Conferencias Interamericanas* sobre *Educación Matemática*" [Interamerican Conferences *on* Mathematics Education].

In the VIII CIAEM, the terms "*Conferencia Interamericana* de *Educación Matemática*" and "*Comité Interamericano* de *Educación Matemática*" ["Inter-American Conference of Mathematics Education" and "Inter-American Committee of Mathematics Education"] were used.

Thus, over the 60 years of history of CIAEM and its conferences, the names used have had minor differences. The predominant ones so far have been as in the VIII CIAEM. The different ways of recording the names of the conferences and the committee were due to the participation of many people or entities (from diverse countries and languages who participated in the processes of organizing and recording the events and materials derived from them). Sometimes they were errors of translation, and sometimes of interpretation. What persisted intact were the acronyms CIAEM and IACME for both the conferences and the committee.

## The Case of Costa Rica

In 1961, Bernardo Alfaro-Sagot participated in the I CIAEM. He was the Director (1959–1964) of the Department of Physics and Mathematics at the University of Costa Rica (UCR), the only university in Costa Rica at that time. He was influenced by the reform purposes and in particular the contact with SMSG textbooks. In fact, in 1964 he published his two-volume book, *Modern Course in Mathematics for Secondary Education*, with the support of SMSG and guidelines of the conference (Barrantes and Ruiz 1995). Notwithstanding, it should be noted that Alfaro-Sagot did not fully follow the New Math guidelines. He professed a great enthusiasm for Euclidean geometry and physics.

Between 1962 and 1963, the Ministry of Public Education of Costa Rica developed a new curriculum for all secondary education, and Alfaro-Sagot led the elaboration of mathematics. In December 1963, the Higher Council of Education of Costa Rica (the main Costa Rican educational authority) approved the new curriculum (which was clearly influenced by New Math ideas). It has been implemented since 1964) (Barrantes and Ruiz 1995).

### *The Implementation of the Reform*

Costa Ricans followed the guidelines of the CIAEM, almost to the letter, with respect to curriculum reform recommendations, textbooks, and actions for initial and in-service teacher preparation. Once the curriculum was reformed and the textbooks were written with the new approach, few modifications were made over the following decades with respect to the mathematics content and its teaching in the country. Changes began to occur only in the 1980s.

In 1967, the UCR began an initial preparation program for secondary school teachers in mathematics. Although it included in some way the New Math, did not fully correspond to that perspective.

The role of the UCR was always significant in the reform.

- Three of the members of the writing committee for the 1964 curriculum were professors of the department, and two of them (José Joaquín Trejos-Fernández and Alfaro-Sagot) had been Directors of the department.
- Alfaro-Sagot attended the I CIAEM when he was the Director of the department.
- The two Costa Rican delegates to the II CIAEM were professors of the department (one of them, Henry McGhie, was, at that time, its Director, the other was Bernardo Montero).
- A new initial preparation program for secondary school teachers at the UCR was introduced in the middle of the reform.

What happened in that institution would affect all the teaching of mathematics in the country. As we said before: The role of universities in the reform was a key factor within the Latin American region.

**Scholars, Students, and the President of Costa Rica** In its early years, the Physics and Mathematics Department hired Olgierd Alf Biberstein (1959–1961). Biberstein was born in Poland and studied in France, with mathematicians like Charles Ehresmann, André Lichnerowicz, André Roussel, and Henri Cartan. In Costa Rica he introduced many new higher mathematics directions, always embracing a formal abstract mathematical vision. Among Biberstein's students was Enrique Góngora, who would become CIAEM secretary between 1972 and 1975 (CIAEM-IACME 2021; Herrera 1995). Another of Biberstein's students was Francisco Ramírez who became the first Director of the Department (or School) of Mathematics in the 1970s (Herrera 1995; Ruiz 1995).

José Joaquín Trejos-Fernández was a person connected to Marshall Stone, although not through CIAEM. He studied mathematics at the University of Chicago where he was in contact with Stone. He

would become very important not only in the UCR but also in Costa Rica as a whole. He was Dean of the *Facultad de Ciencias y Letras* [Faculty of Sciences and Humanities] of which the Department of Physics and Mathematics was part. Trejos-Fernández was President of Costa Rica between 1966 and 1970 (Ruiz 1995).

During the last three decades of the twentieth century, Latin America produced mathematics and associated fields graduates from US and European universities who had an impact on mathematics education within this region. We can mention Carlos Vasco, Ricardo Losada, and Luis Carlos Arboleda (Colombia), Eduardo Luna (Dominican Republic), Eugenio Filloy (México), Jaime Michelow and Fidel Oteiza (Chile), and Tania Campos (Brazil).

## *A Second "Breath"*

In the 1970s the Department of Mathematics at the UCR was created—although from 1974 it would be called a "School." The first Director was Ramírez (1972–1974), who had studied in France at a time when the Bourbaki group was still very influential. He initiated the hiring of many foreign professors, some of them French (Ruiz 1995). But during his administration, the New Math was not particularly promoted. That changed in the following years.

Under the influence of Montero, as Director of the Department (1975–1978 and 1983–1986), what might be called a second "breath" for New Math took place. Montero had attended the II CIAEM as one of the Costa Rican representatives. His administration would have a strong impact on university preparation programs for both mathematicians and high school mathematics teachers. Euclidean geometry was weakened, and algebra, vector spaces, and affine geometry were strengthened. Above all, the preparation of professionals was nourished with heavy loads of "formalism" and "purism" required. Unfortunately, though, a negative attitude toward the practice or profession of teaching was promoted with a preference being displayed toward pure mathematics and pure mathematicians (Ruiz 1995).

This influx nurtured a group of professionals who constituted the mathematical community of the country until the first decade of the twenty-first century. During the 1970s, other public universities were created in Costa Rica, and alternative programs for the preparation of mathematicians and mathematics teachers were developed. However, as pointed out by Ruiz (1995), many of the "formalist" or "purist" ideas of those associated with the New Math continued to influence these programs, although the intensity varied across institutions. For decades, the teaching of geometry was weakened in such a way that many teachers chose not to teach it, with a "formalist" vision of algebra, relations, and functions being preferred, with much emphasis being placed on mechanistic practices.

Ruiz (2018) has argued that, with some exceptions, in the 1970s and 1980s, Costa Rica did not promote intellectual visions in line with international trends in mathematics education. Mathematics was not presented as an independent professional and scientific discipline.

## *Breaking with the New Math*

A first strong rupture with the New Math in the national curriculum of Costa Rica occurred in 1995, with several weaknesses associated with the reform of modern mathematics being progressively eliminated. What was most relevant with respect to the curricular changes in 1995 was that a *constructivist* perspective was formally adopted (MEP 2012; Ruiz 2020a). However, there was no connection between the foundations, supposedly constructivist, and the syllabuses for each educational level, which were basically lists of mathematical content. Additionally, a *behavioristic* emphasis was

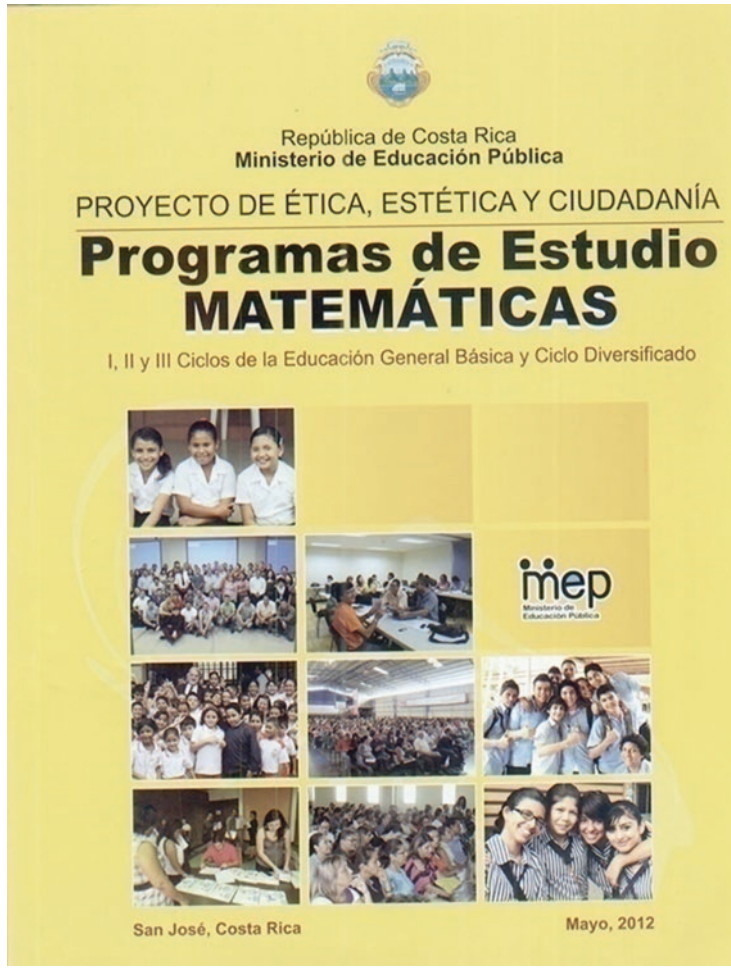


*Figure 19.4* Cover of 2012, 1–12 School Mathematics Curriculum approved by Costa Rican Education Authorities

evident in almost all of the intended curricula. This had always prevailed throughout the twentieth century irrespective of any New Math emphasis being present. The 1995 curriculum was altered in 2001 and 2005, but its essence was not changed (Ruiz 2018).

As Ruiz (2020b) pointed out, in 2012 the mathematics curriculum underwent a real revolution, a "quantum leap." A new curriculum (MEP 2012) was adopted for all pre-university education (except preschool) which not only took account of the results of practices and research from the international mathematics education community but also made its own original theoretical contributions which were adapted to national contexts (see Figure 19.4). For example, the new curriculum emphasized that higher cognitive capabilities should be recognized and applied in all mathematical study, and "general mathematical competence" was to be conceived in pragmatic terms, with an articulated syllabus being based on mathematical areas and learners' abilities (Ruiz 2013a, 2015, 2018). Another feature is that "problem solving" was interpreted as a pedagogical strategy: A central focus was that each lesson should start with a problem which had a *strong emphasis on real contexts*. To this central focus, the new curriculum also called for the use of technologies appropriate for different students in different social scenarios. It incorporates, too, the use of the history of mathematics. An important explicit aim

is to cultivate positive attitudes and beliefs about mathematics and its teaching. More than 1700 "*indicaciones específicas"* [specific indications] and examples are provided *within the curriculum* to support each teacher's understandings and classroom practices.

The 2012 curriculum formally completed the rupture not only with New Math but also with other curricular perspectives that influenced mathematics education in Costa Rica over many decades. The implementation of this ambitious curriculum has been full of important innovations, including strong use of communication technologies as curricular instruments (Mathematics Education Reform in Costa Rica Project 2021; Ruiz 2018, 2020a). Notwithstanding, some of the "ghosts" of previous ideas and curricula still arise as obstacles to the new reform.

While the New Math reform was led by professional mathematicians, the new reform was led by mathematics educators (Mathematics Education Reform in Costa Rica Project 2021). That is a significant difference.

In Costa Rica during the period of the New Math one university had a major influence on how mathematics was presented in the schools. In the new reform environment a small number of researchers from the same university and other public universities now played a central role, but this time only individually (and not institutionally). The universities have had to adjust their initial teacher preparation programs to match the new official curriculum.

### *Costa Rica and CIAEM*

In the 1960s, the teaching of mathematics in Costa Rica was much influenced by the CIAEM. Half a century later, during the second decade of the twenty-first century, the CIAEM maintains an influence and receives significant support (e.g., in the organization of the conferences) from the professional team which developed the reform in that country (Mathematics Education Reform in Costa Rica Project 2021). Since 2007, the leader of this reform has been the president of CIAEM. There is a symbiotic positive relationship.

The presence of Costa Rican scholars on the CIAEM executive committee since its creation in 1961 should be mentioned: Alfaro-Sagot was a member of the *pro tempore* committee in 1961–1966, Góngora was secretary in 1972–1979, and Ruiz was secretary in 1987–1995, vice-president in 2003-2007, and president in 2007–2023 (CIAEM-IACME 2021).

## Concluding Remarks

In the second half of the 1970s, New Math reform in Costa Rica entered a period of crisis. Serious questions began to be asked about the reform and a new scenario for the teaching of mathematics emerged. Notwithstanding, in the world and within the Latin American region not all countries dissociated themselves in the same way or with the same speed from the ideas and perspectives of the New Math. Costa Rica is a country where, in the mid-1970s, such views still prevailed. It was another 20 years before a clearly different kind of curriculum was approved (and formally identified with the label "constructivist"). It would take a further 20 years for a curriculum based on international mathematics education research and experiences to be officially approved (MEP 2012). Brazil took a different pathway: In the mid-1970s, different perspectives were adopted in Brazil with sociocultural factors being emphasized, thanks largely to the efforts of Ubiratan D'Ambrosio, and later still the use of modeling was given special attention (D'Ambrosio et al. 2015).

Despite its inadequate premises and, arguably wrong, theoretical elements and objectives, the New Math reform generated educationally important results for both mathematics and mathematics education in Costa Rica. Among these results were the following:

- The place occupied by mathematicians in the university, which had often been questioned by others, was strengthened. The banner of mathematics reform contributed to the development of higher mathematics communities.
- The ties between mathematicians in Latin America, the USA, Canada, and Europe became much closer. Before the reform, these ties were more isolated or had relevance only in certain countries (e.g., Argentina, Brazil, Mexico), but with the reform, these associations and collaborations in and with many other countries were developed.
- The reform in the region motivated professionals who were involved in the teaching of mathematics. The universities sought to strengthen a social base for the independent development of mathematics education. Many of the scholars who participated in the reform whose training was in mathematics began to take their teaching more seriously than ever before. Intersections between the communities of mathematicians and mathematics educators were developed.
- There was progress toward a different "balance" between pedagogy and mathematical discipline. Previously, the objectives of mathematics teaching had been in the hands of pedagogues with little knowledge of mathematics. The reform helped to provide a path that promoted a pedagogy of mathematics, and that continued to develop within the initial preparation programs for mathematics teachers (although sometimes very slowly).

What happened in the Americas with the main international agents of the New Math? In 1961, with the CIAEM and its conferences, an intellectual and organizational space was created. This served as a valuable North-South and South-South bridge and served to empower the mathematics communities and their teaching within the region. And this development has continued since then, though with important transformations: 15 conferences had already been held during 1961–2019 (CIAEM-IACME 2021).

We have reviewed here, with special emphasis, the effects of the first four conferences, and then the distancing from the New Math that was forcefully sanctioned at the Campinas conference (V CIAEM). This transformation was related, on the one hand, to the evolution that mathematics education was experiencing worldwide and, on the other hand, to the demands that Latin American societies were making. As we saw, in 1979 a second stage in the history of the CIAEM and its conferences began.

Finally, it should be noted that since 2007 a third stage has developed. This new phase actively uses modern technologies, promotes high scientific quality in its events, and continues a vision of support for regional communities but with an internationalist and global perspective (Sánchez-Fernández 2019; Scott 2015). Before 2007, in organizational terms, the CIAEM was essentially a "committee" comprising an executive committee and some national representatives, but since 2007 it has been conceived in another way: It became a flexible "community." In its "terms of reference" it is now defined as an "International Community, in the Americas, of researchers and educators in Mathematics Education with solid scientific and educational links to the rest of the world" (CIAEM-IACME 2021). It forms a broader *network*, which includes other actions in addition to the conference—translations, local scientific events, social networks, various publications, blogs, a virtual community, etc. CIAEM has developed strategic relationships with regional organizations: The *Red de Educación Matemática de América Central y El Caribe* [Mathematics Education Network for Central America and the Caribbean] (since 2012), Mathematics Education Reform in Costa Rica Project (since 2012), and the Mathematics Education Community of South America (since 2016). CIAEM, for example, has been very important in assisting development within the region ICMI's Capacity and Networking Project (Sánchez-Fernández 2019).

As has happened in many countries, in Latin America regardless of the ideas that were embraced by its promoters, the New Math was an educational and social movement that served as an engine for supporting ideas and practices within the mathematics communities and their teaching. Sometimes, new agents and movements, ideas and perspectives, resources, and individuals appeared. Sixty years

after its foundation, CIAEM and the conferences and the diverse actions and strategic relationships it has propagated, continue to offer avant-garde means which promote the progress of mathematics education in this region.

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