ANGEL RUIZ

4. TECHNOLOGY AS A CURRICULAR INSTRUMENT

This chapter discussed the introduction of technological objects within a school mathematics curriculum and during its implementation. The focus is on the nature of technological tools and processes that were transmuted by a specific social and national context, in this case a small peripheral and developing country; Costa Rica. This case provides a basis for comprehension of the nature of these objects as social means; a perspective in which appropriate formulations and modulations are relevant and crucial for real social and educational action, and not just for academic purposes. Addressing the connection of technology tools and processes to mathematics curriculum development and implementation within this Costa Rican experience is intended to shed some light on this perspective and the connection to mathematics teacher education.

INTRODUCTION

The introduction of technological objects within a school curriculum and during its implementation is just a premise in the scenario we live in. However, the way this introduction is conceived and effectively developed is something that cannot be seen as "tools and processes" that can be applied equally in all nations. Sometimes there are local or regional defined protocols for curriculum design and implementation, sometimes not only do they not exist, but the curricular actions depend on the day-to-day within an unstable educational community. The specific social, cultural and political fabric intervenes and so does the particular historical moment.

In the recent experience in Costa Rica, technological contents have been included within the written intended curriculum, and there have been innovative and cutting-edge results in the use of technology tools to implement this new curriculum, including blended courses, MOOCs (Massive Open Online Courses) and what the reformers have denoted Mini-MOOCs, both for teachers and students. However, the most interesting thing here is not these tools in themselves, because these are used in many countries, what is special is how these "objects" have responded in a direct way to the needs of a reform in the teaching and learning of mathematics that began with the curriculum formally approved in 2012. One central question was: which technological "objects" were relevant and how to modulate the scope and use of

© KONINKLIJKE BRILL NV, LEIDEN, 2020 | DOI: 10.1163/9789004418967_005

them in connection to curricular aims for a feasible achievement in the classrooms? There is thus a desirable dialectic between intended and achievable curriculum.

What does all of this imply? Clearly, the nature of these technological tools and processes were transmuted by a specific social and national context, in this case a small peripheral and developing country. This happens because technological objects cannot be seen as *in vitro* beings; where even the social "mediation" normally associated with such tools (in the theory), can be too abstract and incomplete to allow comprehension of the nature of these objects as social means. In addition, appropriate formulations and modulations here are relevant and crucial for real social and educational action, and not just for academic purposes. Somehow, the connection of technology tools and processes to curricular development within this Costa Rican experience may shed some light on this perspective. This is the aim of this chapter.

Inevitably, it is necessary to describe some of the general characteristics of the teaching of mathematics in this country, the social and cultural context, additionally to summarize the central points of the curricular change, its general implementation, and in particular to recognize the academic and human agents who have oriented and developed this process.

One caveat, the curriculum concept can be considered in many ways: for example, Kilpatrick (1994) notes that "The curriculum can be seen as an amalgam of goals, content, instruction and materials" (p. 7). Niss (2016) establishes six components that define curriculum: goals, content, materials, forms of teaching, student activities, assessment. We agree that those components should be considered when we refer to curriculum matters, but do not necessarily agree that the concept "curriculum" should embrace all of them. In this work curriculum will be understood essentially in relation to goals and contents, and assessment.

EDUCATION AND MATHEMATICS IN COSTA RICA

Costa Rica always gave special importance to education: a nation of "more teachers than soldiers." In 1949, this country constitutionally abolished the army and a good part of the resources allocated to the army were transferred to education (Trejos, 2018). General indicators such as literacy and educational coverage are high (OECD, 2017). Undoubtedly, Costa Rica's results in international comparative tests are better than all of the Central American region, and among the best in all of Latin America, as reflected in the *Second and Third Regional Comparative and Explanatory Studies* (SERCE and TERCE) of UNESCO's *Latin American Laboratory for Assessment of the Quality of Education* (see Latin American Laboratory for Assessment of the Quality of Education, 2014); and the same in relation to the *Program for International Student Assessment* (PISA) tests of the *Organization for Economic Co-operation and Development* (OECD). However, the results of this entire region are in the lowest percentiles of the countries that administered the last test (OECD, 2010, 2014, 2016).

Despite its regionally good educational performance, there are problems in the teaching of school mathematics, which are reflected in several ways. For example, the results in the national examinations that are administered to students completing secondary education (a requirement to enter higher education) show scores in mathematics at an average of 15–20 points below those of the other subjects. On the other hand, the best universities in the country, which are public (see Consejo Superior de Investigaciones Científicas, 2018), perform diagnostic tests for students from secondary education who wish to study university programs that require mathematics (see for example University of Costa Rica, 2018). For more than 10 years, only slightly more than 15% passed these tests. A related issue is that there are few students who study for Science, Technology, Engineering, Mathematics [STEM] careers at the universities, something that weakens the nation's ability to be able to carry out actions that strongly rely on sciences and technologies.

Mathephobia

One of the factors associated with all these structural weaknesses in student performance and the choices of university studies is what can be called "mathephobia" (Papert, 1980; Buxton, 1981; Maxwell, 1989). That is, a sociocultural syndrome of fear, anxiety and rejection of mathematics, something that may start before school. In the origins of this syndrome are at least two factors: the absence of adequate pedagogical strategies that allow generating the supports and scaffolding that the abstract nature of mathematics requires, as well as the constant of low performances that causes low self-esteem in students. Student self-efficacy has been shown to be a factor clearly correlated with performance (Williams & Williams, 2010; Goldin et al., 2016). There are beliefs about mathematics that are implied here, like those that affirm that this science is only for geniuses. Or that if a problem is not solved in a few minutes it is because it is impossible for the student.

All these things, related, pose formidable challenges for classroom action and mathematics teacher preparation, but also for the school curriculum.

Teacher's Preparation

In Costa Rica, the initial preparation of teachers is carried out in universities. In the case of primary school teachers, the programs essentially offer a "generalist" preparation (to teach many subjects), and, it should be mentioned, there is little mathematics. The initial preparation of secondary school teachers includes mathematics, pedagogy and other subjects in different ways, and, in general, they have weaknesses, especially in terms of the little emphasis given to the specific "pedagogical content knowledge" in mathematics (Shulman, 1986, 1987).

It is also a serious problem that most of the graduates (around 75%) of these initial preparation programs come from private universities (over 50 institutions).

Unfortunately, most of them do not offer good academic conditions to ensure high quality professional performance in the classroom.

While there are processes for university accreditation, these are not mandatory, and are oriented to mostly formal aspects. And to this it must be added that the system of professional recruitment by the main employer of teachers, which is the Ministry of Public Education (MEP), has great weaknesses, because, for example, no entrance tests are required. These conditions affect the educational results that are achieved.

The Social Context

The "developing" conditions of the country are manifested in different ways. To start with, since the 90's there has been a persistent percentage of poverty (around 20%) and above all social inequalities (Gini coefficient around 0.514: Leitón, 2017), large differences between urban and rural areas, and the existence of economically, socially and culturally marginalized populations.

Although a high percentage of the Gross National Product is allocated to education, public action is inefficient, and there is no effective teacher supervision system and little accountability. Government changes that occur every four years can easily reverse positive results from a previous administration. There are important deficits in strategic public policies. Teacher unions have eminently short-sighted visions and little association with the purposes of institutional or national progress and they have considerable strength in imposing their interests (in the past years each government has had to adapt many of its policies to these strong groups).

In addition, Costa Rica, being a small country (surface area of 50,100 square kilometers, around 5 million inhabitants), is very vulnerable to international economic changes. In 2010 there was nothing that could anticipate the development of profound changes in the teaching of mathematics in this country, despite the obvious weaknesses that existed for decades. What happened?

THE REFORM OF 2012

Three years have been decisive in the history of school mathematics curricula in the country: 1964, 1995 and 2012.

New Math

In 1964 the teaching of mathematics underwent a formidable change caused by the "New Math" reform, with a new curriculum and programs for teacher preparation and didactic resources. Its perspective included a strong use of set theory, early introduction of algebraic structures, weakening of Euclidean geometry. As was common in many countries, it was a specific reform in school mathematics content, under the influence of mathematicians. In the case of Costa Rica, as in all Latin

America, with the direct influence of Marshall Stone and the Inter-American Committee of Mathematics Education (Barrantes & Ruiz, 1995; Ruiz, 2014).

1995–2012: Constructivism?

No modifications were seen until 1995 when a constructivist perspective was used, albeit in a very general way. Though in 2001 and 2005 some changes were made in some general curricular dimensions, in mathematics contents and its teaching this curriculum remained intact.

In Costa Rica the new perspective did not turn to "back to the basics," something that happened in other places. This curriculum represented a clear separation from the "New Math" and tried to offer and an alternative to what dominated for over 30 years. However, it was far from the findings and lessons of international mathematics education, for example in the role of problem-solving, statistics and probability, dynamic geometry, the use of technology to deal with mathematical objects and new educational environments, and moreover in the specific pedagogy of mathematics contents. This new curriculum exhibited a very wide distance between its general foundation (with a constructivist facade) and the rest of its elements: syllabi, methodology and assessment. Linear and behavioristic views on curriculum dominated.

The theoretical weaknesses of this curriculum made it impossible for it to be seen by the educational community as anything more than a list of mathematical contents (Ministry of Public Education, 2012; Ruiz, 2017b). It should be noted, however, that in Costa Rica all school curricula for all subjects had the same problems.

Main Concepts of the 2012 Curriculum

In 2012 there was a real revolution in the school mathematics: a new curriculum for all primary and secondary education. The new perspective no longer emphasizes content as had been the case with all the previous curricula and positions itself within recent international trends in education that encourage the use of skills or competencies (Niss, Bruder, Planas, Turner, & Villa-Ochoa, 2016, pp. 237–245). The approach in Costa Rica uses the concepts of *knowledge*, *abilities*, *processes* and *competence*. *Knowledge* is presented as mathematical objects (concepts and procedures that are organized in five content areas: numbers, measurement, geometry, relations and algebra, and statistics and probability). *Abilities* are capabilities that directly refer to that knowledge and are to be developed in short periods of time ("specific") or in educational cycles of 2–3 years ("general"). Five *processes* are transversal actions to the five content areas that seek to generate cognitive abilities: *to reason and to argue, to pose and solve problems, to connect, to communicate, and to represent*. Mathematics *competence* is defined as the general purpose of mathematics preparation in schools for which *knowledge*, *abilities* and *processes* are instrumental.

Competence is interpreted as a general capability to understand and use mathematics in different contexts, but above all it emphasizes the cognitive capabilities developed through the learning of mathematics. The definition of competence used by PISA of the OECD is explicitly accepted; as well as the concepts of "reproduction," "connection" and "reflection" that are used to identify three levels of complexity in mathematical tasks (indicated as competency "clusters" in PISA's 2003 theoretical framework). Despite having used these inputs, it cannot be said that this curriculum was "inspired" by PISA (as was reported by Niss et al., 2016, p. 244). In the theoretical foundations of the Costa Rican curriculum there is a central influence of ideas developed by researchers from that country since the 1980s (Ruiz, 1987, 2000, 2003).

It should be noted that a special distinction is made between "processes" (collections of actions) and transversal higher-order cognitive capabilities. The reason for this differentiation was to emphasize what is or should be done in the classroom. Additionally, as pointed-out by Niss (2003, 2015), higher-order competencies have inevitable intersections, and this is the approach the Costa Rican curriculum has adopted.

There is a special intellectual elaboration that integrates ideas of the National Council of Teachers of Mathematics (NCTM) and PISA in a *suis generis* way. The five higher-order capabilities of the Costa Rican curriculum are related to the five processes that the NCTM indicates (although there are differences). Ministry of Public Education (2012) states how with these capabilities it is possible to develop all the competencies or fundamental capabilities indicated by PISA.

HIGHER-ORDER CAPABILITIES AND PROBLEM-SOLVING WITH EMPHASIS ON REAL CONTEXTS

The new Costa Rican curriculum is quite different from other international experiences that use competencies. They are not the curricular objects from which knowledge (and abilities associated to them) are organized, rather knowledge and associated abilities are the point of departure. It is, then, in the "pedagogical mediation" that building/developing mathematical higher-order capabilities is proposed. That is to say, the curricular vision that was adopted in Costa Rica does not structure its contents and objectives assuming any of the two extreme positions that have dominated internationally: "by content" or "by competencies." Why? Because within this national context it would have been much more difficult, if not impossible, to ask the teachers to manage their classroom action using the competencies as organizational curricular objects. Mathematical content and knowledge provide a more adequate reference to use. Education officials, students and parents agree with this stance.

How then it is proposed to generate higher-order cognitive capabilities? It is through a collection of strategies that integrally allow the construction of those capabilities. One of the most important is an appropriated design of the "pedagogical mediation." A precise approach was proposed to guide the teaching action, and to allow a relatively uniform development throughout the country. It was considered not possible to only offer general orientations and to trust in the preparation and teaching expertise in a country with serious limitations in the educational agents.

Problem-Solving

The approach or focus was called "problem solving," although it was not conceived of in the same terms as the problem solving in Polya (1945) and Schoenfeld (1985) where, for example, heuristics and creativity can play a key role (Liljedalhl, Santos-Trigo, Malaspina, & Bruder, 2016). Here "problem-solving" means the design of mathematical tasks that, based on the cognitive purposes established by the curriculum, allow learning to be triggered and higher-order capabilities to be developed. The designers of the curriculum established a four-step model of classroom action to develop that strategy: (i) posing of the problem, (ii) independent student work, (iii) collaborative discussion and communication, and (iv) closure (Ministry of Public Education, 2012). In particular, an interpretation of the Japanese lesson study was invoked (Shimizu, 2007, 2009; Clarke, Emanuelsson, Jablonka, & Mok, 2006, Clarke, Keitel, & Shimizu, 2006), as well as the use of some theoretical elements from the French School of Didactics of Mathematics, and some results raised in the North American discussion on problem solving (Ministry of Public Education, 2012).

Why call it "problem-solving," terms so general and subject to many diverse interpretations and even distortions? Again, because of the call of reality. This formulation was less conflictive for the authorities that had to approve the curriculum and for the educational community: who would disagree with problem-solving? Something different would have happened if terms such as "competencies" had been strongly used. Within the curriculum "problem-solving" is used as this pedagogical model but also as a higher-order capability.

Real Contexts

To the "problem-solving," an emphasis on real contexts was added This was synonymous with "mathematics for life," something that everyone would agree on. The reformers felt it necessary to design an "image" of the curriculum that would allow its proper "marketing."

Working with real contexts, however, did not obey only to a political strategy, but also an epistemological vision about mathematics and its teaching. In this approach, the real contexts and mathematics modeling play a fundamental role in two senses. On the one hand, they allow an approach to the student environment in which the educational action is carried out, and, on the other, they seek to invoke elements of the mathematical construction in a general way. Of course, there are epistemological premises here about the nature of mathematics and the teaching of mathematics. It is possible to observe here some influence of the "Realistic Mathematical Education" initiated by Freudenthal (1983, 1983, 1991), but above all by Ruiz (1987, 2000, 2003). Some international research results show a clear positive impact of the use of modeling problems, in connection to student self-efficacy (Goldin et al., 2016).

The Synergy of the Curriculum Emphasis

The curriculum explicitly affirms the cultivation of positive attitudes and beliefs about mathematics and its teaching as an important emphasis. The same applies to the use of technologies and the use of the history of mathematics.

History is introduced not only to provide strategies and methodologies for the classroom, but also to create a vision of mathematics and its teaching that will support learning and educational purposes. In order to progress in mathematical learning, it is fundamental to generate a "productive learning behavior" that is:

> a social construct formed from the interaction of learners' personal learning states and mathematical dispositions, their home community, their classroom or learning environment community, and macro-cultural constraints such as curriculum, assessment, and cultural attitudes. (Goldin et al., 2016, p. 18)

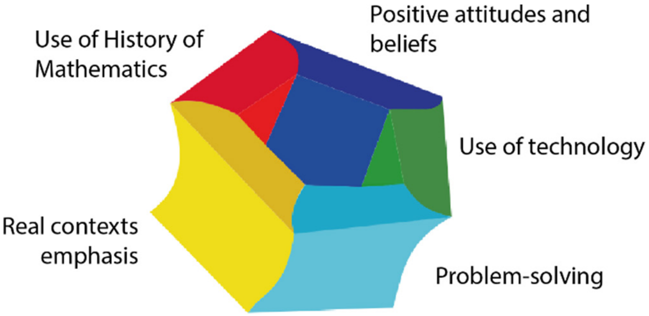


Figure 4.1. The five emphases of the school mathematics curriculum in Costa Rica

All the elements of the Costa Rican curriculum (see Figure 4.1) act in a synergistic way to achieve a "productive learning behavior": a student's engagement in the building of their learning and the development of higher-order capabilities. Using mathematical tasks of different and increasing complexity supports that engagement (Kong, Wong, & Lam, 2016); and the use of real contexts and modelling provide support in that direction. Thus, there are for student independent work. The use of history and technology is associated with this aim.

The Locus of the Mathematics Curriculum

Because in Costa Rica the education system is centralized, the curriculum is mandatory for the entire country. This gives it a relatively greater role than in decentralized systems, however a large number of educational agents have not always used the national curriculum as a central reference. For example, initial teacher preparation programs, professional development actions, and didactic resources did not revolve around the official curriculum. That was partly because until 2012 the curricula were reduced essentially to lists of contents with ancillary curricular foundations. Since 2012 it has been sought that the national curriculum is the heart of the strategies and educational resources in mathematics for all the protagonists involved, the point of reference.

At the end of 2016, the Costa Rican educational authorities approved a new Curricular Policy for all subjects (Ruiz, 2017b). It is in tune with the perspectives of the mathematics curriculum in terms of 21st Century abilities, capacity building and competence. Until that moment, the mathematics curriculum had been a "lone ranger" in the Costa Rican curricular spectrum. And that explains in part some elements of the social and political context in which the implementation of that curriculum has taken place.

THE SOCIAL/POLITICAL SCENARIO

What was the political, social, educational environment in which the new curriculum was approved? This context is important in understanding the curricular design as well as its implementation. It should be noted that the authors of curriculum proposed to the education authorities a first version (issued in the second half of 2011) that was submitted to public universities, unions and other educational sectors. In April 2012, a second final version was presented that included many of the suggestions of the various education actors in the country.

It should be said that despite the incorporation of all these elements and important adjustments to the first version, the new curriculum was rejected openly by teachers' unions and most of university departments that teach mathematics and, besides, there was a rejection, mostly underground, from some middle-rank officials of the Ministry of Public Education. Strong arguments of a technical or intellectual nature against the proposal were not put forward. In several cases it was criticized that the curricular elaboration was not done through a broad national commission that included representatives of all sectors, unions, the universities, officials of all educational regions and several teachers in consultation, in summary: that the process was not democratic. This was the main complaint. And also, it was argued that it was necessary to wait for changes in university prospective teacher preparation programs to include the ideas of the new proposed curriculum, before approving it.

During the following years the opposition declined within the Ministry of Public Education, and the universities have had to adapt their programs to the new official curriculum. Nevertheless, some negative groups have continued to attempt to weaken some of the actions to consolidate the new curriculum, particularly in some regions. During the period 2012–2018, it was never certain that a point of no-return had been reached, the curricular text has always been under siege. Especially in the change of national government of 2014 these groups conspired for the new minister to reverse the 2012 curriculum. That not only did not happen, but the political support for the reform increased. With the next governmental change in 2018, it does not seem possible a return to the past. So far, three ministers have supported this reform. But this politically necessary condition for the curriculum implementation is not sufficient, as we will see in the following sections.

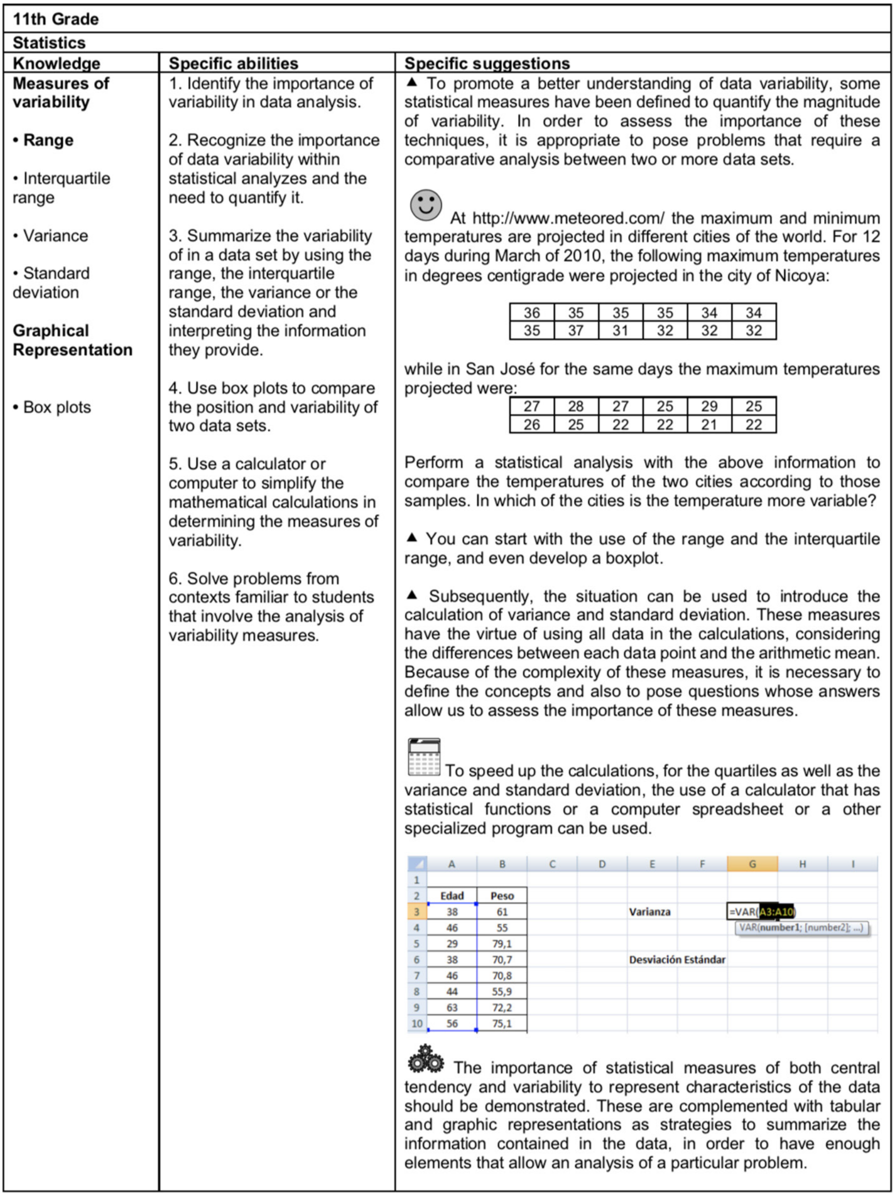
HOW TO IMPLEMENT THE CURRICULUM?

Although the majority of those who wrote the new curriculum were university scholars, they kept in mind what Ruiz (2013) has called a "perspective of praxis in mathematics education." That is to say, the awareness that a curriculum could not be conceived of as a theoretical object to be designed *in vitro* that then had to be implemented. From the beginning it was designed in a way that allowed it to be approved politically and socially, and then as an object to be implemented in the classroom. If a curriculum is not implemented in the classroom, it does not make sense. This vision was instrumental: What knowledge was to be included and what was not? How far should you go with each skill or ability? What should be the curricular emphasis? Which terms were convenient, and which were not? What images were "clever" to present?

Many of the curricular objects were placed to be modulated in the classroom action. This is something that always happens in any country, but when there are very strong internal differences, the range of modulation must be very broad. For example, between being able to use computers and the internet or not being able to do so. A consequence of this reality is that some curricular goals could not be mandatory.

The reformers also left unwritten goals for another stage of implementation. An example of this: no assessment proposal consistent with the new curriculum was introduced initially. Having done so at that time would have been a risk for the same approval of the curriculum within a scenario where behaviorist and linear approaches predominated throughout the education system and, in particular, there were reticent officials in the Ministry of Public Education departments in charge of the assessment. One issue was key, there was awareness of what Niss et al. (2016) states: "Throughout the world there seems to be a lack of adequate helpful guidelines and support for pre-and in-service teachers" (pp. 245–246). When and how should there be a response to the teacher learning challenge?

Given the depth of the curricular changes and given a context with few available educational resources and with teachers with weak preparation, it was necessary to offer support materials with a broader scope than in other types of social realities: right from the beginning. This is why within the same curriculum; more than 1,600 suggestions and hundreds of examples were included to support understanding and curricular implementation (see Figure 4.2). The curriculum itself is not only a collection of contents and general orientations.



11th Grade

Statistics

Knowledge

Measures of variability

- **Range**
- Interquartile range
- Variance
- Standard deviation

Graphical Representation

- Box plots

Specific abilities

1. Identify the importance of variability in data analysis.
2. Recognize the importance of data variability within statistical analyzes and the need to quantify it.
3. Summarize the variability of in a data set by using the range, the interquartile range, the variance or the standard deviation and interpreting the information they provide.
4. Use box plots to compare the position and variability of two data sets.
5. Use a calculator or computer to simplify the mathematical calculations in determining the measures of variability.
6. Solve problems from contexts familiar to students that involve the analysis of variability measures.

Specific suggestions

▲ To promote a better understanding of data variability, some statistical measures have been defined to quantify the magnitude of variability. In order to assess the importance of these techniques, it is appropriate to pose problems that require a comparative analysis between two or more data sets.

☺ At http://www.meteored.com/ the maximum and minimum temperatures are projected in different cities of the world. For 12 days during March of 2010, the following maximum temperatures in degrees centigrade were projected in the city of Nicoya:

36	35	35	35	34	34
35	37	31	32	32	32

while in San José for the same days the maximum temperatures projected were:

27	28	27	25	29	25
26	25	22	22	21	22

Perform a statistical analysis with the above information to compare the temperatures of the two cities according to those samples. In which of the cities is the temperature more variable?

▲ You can start with the use of the range and the interquartile range, and even develop a boxplot.

▲ Subsequently, the situation can be used to introduce the calculation of variance and standard deviation. These measures have the virtue of using all data in the calculations, considering the differences between each data point and the arithmetic mean. Because of the complexity of these measures, it is necessary to define the concepts and also to pose questions whose answers allow us to assess the importance of these measures.

To speed up the calculations, for the quartiles as well as the variance and standard deviation, the use of a calculator that has statistical functions or a computer spreadsheet or a other specialized program can be used.

	A	B	C	D	E	F	G	H	I
1									
2	Edad	Peso							
3	38	61			Varianza		=VAR(A3:A10)		
4	46	55					VAR(number1; [number2]; ...)		
5	29	79,1							
6	38	70,7			Desviación Estándar				
7	46	70,8							
8	44	55,9							
9	63	72,2							
10	56	75,1							

The importance of statistical measures of both central tendency and variability to represent characteristics of the data should be demonstrated. These are complemented with tabular and graphic representations as strategies to summarize the information contained in the data, in order to have enough elements that allow an analysis of a particular problem.

Figure 4.2. The curriculum syllabi in three columns, the third one with specific suggestions (translated from the Spanish)

What about the pace of the implementation? It was gradual, beginning in 2013, to compensate for many years of curricular backwardness. Several annual transition programs were developed, which by 2017 allowed the whole country to work with all of the curriculum at pre-university level (at least in theory). Gradualism was undoubtedly due to the breadth and depth of the changes made, which not only required cognitive sequences in relation to student learning, but especially allowed for opportunities and times for the preparation of practicing teachers.

ONE TEAM FOR BOTH CURRICULUM DESIGN AND ITS IMPLEMENTATION

The same team that wrote the curriculum also assumed a decisive role in the implementation of the reform. This was carried out through a project called *Mathematics Education Reform in Costa Rica* (www.reformamatematica.net, Figure 4.3). The basis of the team were mathematicians and mathematics educators from certain public universities with a long scientific trajectory and important international connections. However, the participation of these scholars was not the result of an agreement with the universities, it was carried out individually. It should be noted that most of them retired from their universities during 2012–2017. The

mini MOOCs

online courses for students and teachers with technology platforms: edX and Moodle.

Read more

blended

Courses for Teachers Who Have classroom sessions and online job

Read more

documents

free documents to support students and teachers.

Read more

Figure 4.3. Frontpage project's website. Three of the main sections: Mini MOOCs, blended courses, documents

project also includes primary and secondary practicing teachers and specialists in communication technologies. Teachers have been provided by the Ministry of Public Education while the specialists and researchers have been hired through financial support from Non-Governmental Organizations.

Between 2012 and 2018, the project's team elaborated, in addition to the transition programs, dozens and dozens of additional support documents with carefully constructed examples that included new curricular ideas, topics, educational emphases, and recommendations for the classroom. All documents were placed online. No other educational curricular reform in Costa Rica has had this amount of resources.

Why were university scholars and private specialists from outside the Ministry of Public Education enlisted in the effort? In 2009 Ministry officials were appointed to write a new curriculum, but they did not succeed. This was not a situation exclusive to mathematics. In October of 2010 Angel Ruiz was appointed by the Minister of Education to lead the curriculum design. The team delivered the first full version of the curriculum in the second half of 2011. Why was such a drastic deadline needed? And, even more important, why was it possible to meet the deadline? Not knowing who the next Minister would be, the reformers tried to secure the new curriculum, as far as possible. Timing was essential. A work as complex as a curriculum with these characteristics was possible only thanks to the existence of these researchers who in some cases had been working in mathematics education for more than 20 years.

Of course, "importing" an external team to the Ministry of Public Education involved from its inception a negative reaction from some officials in the middle and upper ranks. The political will of the highest ministerial authorities has been decisive for the course of this reform.

It is crucial to understand that the written curriculum was just a first step of a deliberately unfinished task. In Costa Rica, unlike other countries, we are not witnessing something like a fixed textbook series that is implemented well or badly, but a living object that must be nurtured and developed to be successful. More than a curricular implementation, one should speak of a long-term radical mathematical reform designed and nurtured by a human team.

It could be said that Costa Rica sought to establish bridges between the results of research and practice, something that specially in the case of competencies (Niss et al., 2016) value as "notoriously difficult" (p. 246). The opportunity to build this bridge was offered by a political context that allowed a new perspective on the teaching and learning of mathematics to be cultivated nationally. The fact that it was also a mandatory national curriculum offered greater opportunities, but also major challenges.

A final comment: The nature of this human team has always implied governmental political support and financial support from Non-Governmental Organizations or private sources. In the past years it has not always has been possible to fund the work of the scholars and technology experts. The situation became critical since 2017, obliging several of these professionals to work for free. To the political uncertainty must also be added the one associated with private financing for experts who can develop actions that would not be possible within the Ministry of Public Education: uncertainty squared.

This long preamble allows us to understand some characteristics of the introduction of technology with this educational reform. Let's begin with the curriculum design.

WHY/HOW TECHNOLOGIES IN THE CURRICULUM?

On the one hand, the use of technologies establishes bridges with generations that are and will be increasingly "digital natives." The perspective is to promote the greater use of artifacts such as computers, tablets, cell phones, global positioning system (GPS), and especially software, platforms, applications, virtual networks. Although the research is not yet conclusive about how much and how the teaching of mathematics improves with the use of technologies (Drijvers, Barzel, Heid, Cao, & Maschieto, 2016), without a doubt, everything points to learning environments where technology and, in particular, virtual technology will participate with greater force.

On the other hand, technologies can facilitate the visualization and manipulation of mathematical objects and methods in a way that would otherwise be more difficult or impossible. As noted by Drijvers et al. (2016): "With technologies, the range of mathematical objects and representations available is wider and more diverse; there are many more examples, models, and virtual experiences available to highlight different features of a mathematical relationship or concept" (p. 22). Technologies generate a redefinition of the mathematics that can and should be taught or understood within the education system. This is true both in terms of mathematical objects and the new student-teacher environments and student-student relations that intervene (Leung & Bolite-Frant, 2015). Identifying and conceptualizing these new elements and especially the type of thinking that emerges are challenges for mathematics education (Liljedalhl et al., 2016, p. 23). Technology is more relevant to a curriculum that emphasizes higher capabilities: "One great benefit of using technology is that it can challenge teaching practices that promote mathematics as a subject only focusing on procedures, and foster mathematics as a subject incorporating conceptual understanding" (Liljedalhl et al., p. 25).

Do these considerations provide something new about what is already known in the educational community about the role of technologies in teaching? No. The peculiar situation we find here is that there were important educational sectors (authorities, officials, advisers, teachers) that rejected an intense use of the technologies in the curriculum design, in general arguing that there are weaknesses in technological infrastructure and national inequalities. Arguments were raised: Why computers and the Internet when there are no walls or chalk in certain schools? Also, how can teachers teach something they were not prepared for? What strategy should be followed?

Flexibility is the starting point. The curriculum sustains the use of technology in various educational settings where the differences can be very large due to the developing nature of the country (urban versus rural, rich versus poor, …). All compulsory curricular content can be developed without technology. However, when it is possible to use technology the curriculum offers the possibilities from the simplest (such as calculators) to the most ambitious (computers, specialized software, the Internet). In this decision, not only the material or socio-cultural limitations were taken into account, but also weaknesses in the preparation of the teachers regarding the use of technology. As with higher capabilities, technology has an important emphasis in the curriculum, but that use is considered to be developed within the "pedagogical mediation."

There are resources focused on the curriculum aims. The materials, activities and courses offered by local educational institutions (universities or Ministry of Public Education) had concentrated always on technological generalities without a close association with curricular goals. For example, textbooks/workshops on the use of Geogebra, Excel, or the Internet were without a link for classroom action or pedagogical strategies. This predominant approach was disrupted by the reformers. The technologies aims are now subordinated to pedagogical and curricular purposes. That is to say, in the first place, the knowledge, abilities and higher-order capabilities that are to be strengthened are defined, and then come the technologies that best adapt to these educational purposes. This is a fundamental premise. What caused more demand for technology in this reform were the conditions to support its implementation.

TWO-STAGES NATIONAL BLENDED COURSES

Such an ambitious proposal that, within a developing country, can take many years to implement, requires multiple strategies at different times for educational resources, and diverse actions oriented towards several protagonists; for example, towards authorities, officials of the education system, students, ministerial departments, researchers, universities. But without a doubt, the privileged place has been occupied by teachers. Obviously, the preparation of teachers is everywhere crucial to implement a curricular change, but in Costa Rica there have been "aggravating" elements: the vulnerabilities of initial preparation programs, and the amplitude and depth of the changes consigned by the new curriculum. The only sensible bet to begin with was the preparation of teachers. And this had to be tailored to the national needs.

Given the weaknesses of the preparation of the teachers, the courses not only had to include pedagogical indications or explanations about the curricular objects, they also had to include properly mathematical contents, because this was an important lack. There needed to be a harmonious integration of mathematics and specific pedagogy of mathematical content within this curriculum environment.

Another issue was how to consolidate the initial implementation in the context of the period of the governmental administration? It was always possible that a new Minister would have a different educational perspective, or she or he would not want to support the curriculum and go backwards. A sword of Damocles was on the heads of reformers.

It was essential to reach as many teachers and educational officials as possible in a very short time. It was not feasible to develop a sequence of face-to-face teacher learning sessions throughout the country. How could a team of less than 15 experts (who wrote the curriculum) reach thousands of practicing teachers within such a period? The central idea was to generate a collection of teachers who could replicate the courses throughout the country, turning them into teacher educators or leaders. An intense initial phase was required in which the curriculum experts directly had contact with those selected teachers who had to be the most capable ones in the country (because they were to serve later as leaders). This gave rise to a strategy in two stages. How could an acceptable degree of quality be preserved in the courses given in the second stage? How would it be possible to reach teachers in regions all over the country, some with little access to possible face-to-face contact because long distances, uneasy material conditions or because they have very limited time to spend in professional development activities? Technology became a key ally to respond to all these needs. Between 2012 and 2016, blended *national* courses were designed and executed, with face-to-face and virtual components, the latter using the Moodle platform.

The aim was to include actions in virtual learning environments as: "Access to online media for communication and the imminent viability of alternative forms of delivery, such as virtual learning environments (VLE), enable professional development to be more accessible to teachers to overcome issues of geographical distance" (Drijvers, 2016, pp. 23–24). In particular, this is true in isolated or rural regions where "there may be a small number of mathematics teachers, or where face-to-face professional development is impossible due to distance" (pp. 23–24).

The officials and leading teachers, with the administrative support of the central offices of the Ministry, developed the same course they received in the first phase in all the different educational regions. The documentation, the self-assessment practices, the examinations and all the resources were essentially the same in the two phases. This process allowed a good amount of academic quality for each course in the two phases. Instead of a multiple "sequenced" strategy that is diluted with each step, a strategy of only two phases, articulated by the possibilities that technology allowed, was offered. It was important that during the first stage teachers experienced the courses as they were proposed to be replicated in the second stage.

Why blended? As Watson (2008) pointed out: "This blended approach combines the best elements of online and face-to-face learning. It is likely to emerge as the predominant model of the future – and to become far more common than either one alone" (p. 3). Were the face-to-face sessions of this strategy important? They were important for two reasons. The first was because the country was not – nor is it now – accustomed to virtual learning environments. In both the first and second stages, this contact with teachers was important; especially because it was about implementing a radically new reform. The second reason was because through face-to-face sessions conducted directly by the reformers, it was possible to generate a collection of leading teachers in the reform throughout the country. It was necessary to cultivate an identity and a common purpose.

Each of the courses demanded a dedication of between 40 and 80 hours (approximately half in online work). In this period, the online part included self-assessment practices and examinations; the teaching materials that were offered were documents. Through this two-stages, blended teacher education strategy, it was possible to prepare almost all secondary school mathematics teachers (about 2700), and 50 to 60% of primary school teachers (22,000). This strategy was not only an "abstract" or "neutral" semi-virtual courses modality separated from political and social goals. The aims of the reformers were those that gave meaning to this modality. Thus, this process allowed a national advancement in the culture of the use of technological and virtual resources by the teaching population even beyond mathematics. The impact was even greater, because the experience in mathematics has become a model to replicate in other curricular reforms within the country.

Since the second half of 2017, new blended courses have been conducted, but not within the two-stages strategy. Instead the blended courses have been aimed at serving specific education regions that have exhibited weaknesses in the implementation of the curriculum. These courses implied a dedication of 40 to 80 hours with a virtual component of between 70 and 80%. A difference in relation to the national blended courses of the period 2012–2016 is that in this new stage the development of the topics is done through videos for the most part (the documents now only serve as complementary support).

In Costa Rica the actions of preparing practicing teachers had been, until this experience, very few, without continuity, and even more reduced in primary education. And with zero technology. Just as the use of technology within the curricular design has been a function of the educational goals, their use in the curricular implementation has responded to the specific needs to carry-out this reform. Were blended courses enough?

MASSIVE OPEN ONLINE COURSES (MOOCS)

There were two circumstances that motivated another teacher preparation strategy. One was that there were still many teachers to be reached. The other was that the preparation provided by the newly generated leaders was uneven across the country (and in many cases had serious weaknesses); it had to be reinforced.

During 2014 and 2015, completely virtual courses were offered as Massive Open Online Courses (MOOCS). This modality is based on the benefits of Internet 2.0 by offering cognitive content through videos and other multimedia elements. As they work with massive populations, the criteria for pedagogical mediation are rethought, as, for example, in relation to certification, the use of social networks, the interaction between teachers and students, and between students and students. The *Class2go* technology platform was used in 2014 and since 2015 Open edX. Each course demanded a dedication of between 30 and 50 hours.

Internationally, MOOCs have been used in higher education. MOOCs in Costa Rica have been associated with this school mathematics reform, and the professional development of the specific population of practicing teachers to support the implementation of the new curriculum. That perhaps explains that in 2014–2015 the levels of retention were in the order of 30% or more, which is well above the international average of 10% (Hone & El Said, 2016; MOOC Maker, 2016).

It must be added, however, that assessing performance in a MOOC in terms of the proportion of those enrolled that complete them is not sufficient: “The definition of completion rate as a percentage of enrolled students may be over-simplistic” (Jordan, 2015, p. 355). So far it has been the usual, but “the potential for more detailed and robust meta-analysis is likely to increase in the future” (p. 355).

Another element of technological innovation within this reform was the design and development of MOOCs for high school students who must take exit tests from the education system to complete their studies and be able to access higher education: national Baccalaureate tests. Based on the gradual process of the curricular implementation, the first national examinations for these students (for the regular high schools, not for the technical high schools) were held in 2016. In 2017 the tests were applied in all high schools. These virtual courses were offered in 2016 and 2017, and about 7,000 enrolled.

International experience indicates that when deep reforms are made in education, during the first years, student performance weakens, and only after a few years do they recover. It is what is called the “implementation dip” (Fullan, 2008). The reformers, the education authorities and the country had to face a formidable challenge here, because it was possible to use weaker student performance as an excuse to attack the new curriculum and go backwards. The results, however, in these two years were not very different from those obtained in the previous 10 years.

As the Costa Rican curriculum emphasizes real contexts (related to the environments of Costa Rican students), many of the mathematical tasks considered in the MOOCs were formulated with real contexts. Although internationally there are a number of virtual resources that explain mathematical objects or knowledge (such as the Khan Academy does), they do not use real contexts, much less those associated with Costa Rican students’ environments. In the same way, there are other elements of the curriculum that made necessary a unique design adjusted to the curriculum: the four-step model for the learning construction strategy, the precise roles of history and technologies.

It was essential to develop high quality digital curriculum resources, tailored to the local needs of the reform, and make them accessible.

MINI-MOOCS

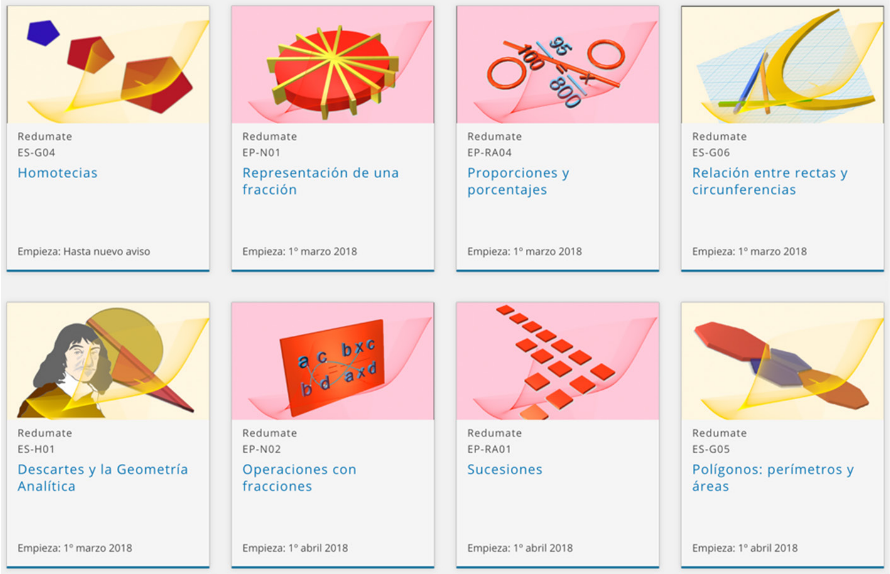
The experiences of the years 2014 to 2016 led to a new innovation: The Mini-MOOCs. These are courses with the same virtues as MOOCs, but with additional features focused on specific, compact, short and self-sufficient topics. Each one can be completed in less than 15 hours. The Mini-MOOCs (e.g., Figure 4.4) are grouped into collections. The idea was to offer virtual units available in a more flexible way for the users. The perspective that has been taken is to create spaces that respond more to individual (personalized) needs. This effort converges with:

> A fundamental shift towards a more open and student-pull model for learning is needed-a shift towards a more personalized, social, open, dynamic, and knowledge-pull model as opposed to the one-size fits all, centralized, static, top-down, and knowledge-push models of traditional learning. (Borba, Askar, Engelbrecht, Gadanidis, Llinares, & Sánchez-Aguilar, 2016, p. 228)

The "modularization" of MOOCs is a direction that has been considered internationally. As Jordan (2015) noted:

> [It] has already been suggested by some (for example, Bol cited in *Harvard Magazine*, 2013; Challen & Seltzer, 2014); the evidence here provides an empirical rationale for such developments, and further research would be valuable to examine the effects in practice. (p. 354)

Also, Hone and El Said (2016) mention that to increase retention in this type of courses it is necessary to adjust the parameters that affect it: the content, the effectiveness of the course that users perceive, and the participation of the facilitators. In Costa Rica, the new modality made adjustments in terms of the first two factors. Between 2017 and 2018, more than 50 of these mini courses were designed and implemented.



Redumate
ES-G04
Homotecias
Empieza: Hasta nuevo aviso

Redumate
EP-N01
Representación de una fracción
Empieza: 1° marzo 2018

Redumate
EP-RA04
Proporciones y porcentajes
Empieza: 1° marzo 2018

Redumate
ES-G06
Relación entre rectas y circunferencias
Empieza: 1° marzo 2018

Redumate
ES-H01
Descartes y la Geometría Analítica
Empieza: 1° marzo 2018

Redumate
EP-N02
Operaciones con fracciones
Empieza: 1° abril 2018

Redumate
EP-RA01
Sucesiones
Empieza: 1° abril 2018

Redumate
ES-G05
Polígonos: perímetros y áreas
Empieza: 1° abril 2018

Figure 4.4. Mini MOOCs (thumbs of eight courses within the edX platform)

The Mini-MOOC modality has been applied to offer courses for both the student population and for teachers. In the case of students, the virtual object is used according to specific needs, as in some cases a user may only need a unit on a specific geometry topic and does not have to enroll in an extensive course where besides the specific geometry topic there may be statistics, probability and algebra from the grade level. All the content you do not need can become a distraction, which can dissuade you from using the virtual object that could serve you at that time. In the case of teachers, their limitations of time as practicing professionals with multiple responsibilities restrict their possibilities to take a long course. In addition, more compact virtual units are easier to use to support teachers' work in the classroom at certain times (when examples and guidance to teach a precise topic are needed). This experience in Costa Rica can provide inputs for the design of MOOCs and individualized modular versions to a certain extent.

Why MOOCs or Mini-MOOCs and Not Just Videos and Exercises?

As indicated by Borba et al. (2016) "In the mathematics education context, MOOCs are 'courses' because there are learning objectives, content and resources, facilitators, ways to connect and collaborate, and at the beginning and end of the learning experience" (p. 25). Precisely because of the specific needs of this reform, it has been necessary to provoke a greater commitment on the part of the users, and at the same time identify the populations that have used these learning environments. In particular, with this type of media and the use of social networks, it was intended to generate a sense of collective belonging that is crucial for developing an innovative curriculum (Korhonen & Lavonen, 2017). This sense of belonging sought to renew the identity of the teachers, and weaken the reactions to change, because as Goldin et al. (2016) affirm:

> Most studies in one way or another demonstrate the interaction between teacher identities and teachers' practices, making it apparent that changing one will affect the other. Moreover, the external demands posed on teachers (e.g., school reforms) inevitably affect teachers' identities: teachers often see changes as threatening to their identity, thus their identity becomes an obstacle for change. (p. 16)

It was necessary to formulate questions, problems, designs and complementary elements to expand the content transmitted through the videos. It was not just about learning for a university course or an individual certification, it was about convincing and strengthening the reform. It was relevant to the extent that interactions were generated among teachers and of teachers with the designers and facilitators of the courses. This was achieved through internal forums on the platforms that were used, but also through a system of consultations-responses with emails, as well as through the use of social networks. On the other hand, the videos did not focus only on a virtual blackboard with the mathematical objects and the mathematical manipulations (drawing of figures, movements, etc.), but they included a visual contact with the writers of the curriculum and the creators of the courses and the participation of different visual and multimedia resources.

Since the second half of 2017, all MOOC-like virtual learning environments are Mini-MOOCs. In 2018 the course offerings were staggered with only 4 in each month for teachers, and 11 for students in two precise periods; thus, adjusting the offerings for the best possible use by teachers and students. The experience until 2018 reveals that, although the Mini-MOOCs have not significantly increased the numbers of people enrolled in the courses, the work completed within the courses by participants has increased.

The Educational Environments Using Technology

The Costa Rican mathematical reform has used MOOCs and blended courses according to its needs, but it is not excluded that in the future MOOCs can be used for smaller populations, for which the use of both Open edX or Moodle platforms can be considered. An example using Open edX can be found in Fox (2017). Since 2016, the reformers have also offered educational materials through Apps for mobile devices.

Also, since the last part of 2018 the reformers in Costa Rica have devoted important efforts to expand their offer of virtual learning environments. In addition to documents, blended courses and Mini-MOOCs have been offered to provide functional virtual educational resources so that secondary students can perform an independent preparation (not only for a national certification examination), and for teachers. The same materials at the same time will provide in a more direct way support for teachers in designing their lessons. They are texts, videos, practices, recommendations, links: open educational resources, that can be accessed without any type of registration in a platform or in a course. This type of open educational resources is not new in the world, but it is in Costa Rica and most of the Latin American region, and besides, it is tailored to the nature of this particular curriculum, and within an integrating strategy that includes Mini-MOOCs, blended courses and a living educational community around the curricular implementation.

Borba et al. (2016) identify four phases in the use of digital technology in mathematics education starting with “Logo,” then dynamic geometry (Cabri or Geometer’s Sketchpad), and a third based on the Internet: “In mathematics education, the way the Internet can be used in a blended learning environment characterizes the third phase, which introduces online courses and new problems” (p. 222). The fourth is with MOOCs, storage and digital interaction in the cloud, and mobile technologies (p. 223). In connection to this the most appropriate thing to say is that in this experience in Costa Rica actions have been carried out somehow with elements of the three final phases, with an intersection between them.

A Final Synthesis

The elements so far considered can be summarized in Table 4.1.

Table 4.1. Design and implementation of the mathematics school curriculum in Costa Rica: A synthesis

Element	*Description*
General purpose of the curriculum	Mathematical competence with a pragmatic vision, and a special emphasis on the cultivation of higher-order capabilities transversal to knowledge.
Curriculum structure	Neither "by content" nor "by competencies," the aim is to develop higher-order capabilities through specific educational strategies.
Basic concepts	Knowledge, specific and general abilities, general mathematical competence, processes.
Classroom action model	Selection and design of problems to build lessons using four steps, and with an emphasis on the use of real contexts and modeling.
Technology in the curriculum	Use of technologies based on precise curricular goals and with the possibility of adaption to multiple educational scenarios.
Starting point for learning and competence development	Engagement of the students in their learning, for which the explicit cultivation of positive attitudes and beliefs is central. There is a synergy of all curricular elements for student engagement.
Curricular implementation	Gradual implementation of the curriculum with the support of annual transition syllabi, recommendations, documents, videos, courses.
Practicing teacher preparation	Combination of two-stages blended courses, MOOCs and Mini MOOCs oriented to use the curriculum in the classroom, as well as to compensate weaknesses in the initial preparation of the teachers.
High school student support	MOOCs and Mini MOOCs to support high school students to prepare for mandatory national school exit examinations.
Technology in the curriculum implementation	Innovative multiple uses of ITC for teacher and student preparation, developing a *Virtual Community of Mathematics Education* through websites, social networks, apps, blended and virtual courses, digital curriculum resources.
The role of social and political factors	A team of scholars committed to the reform has been crucial and has facilitated the establishment of innovative bridges between research and practice. The design and development of the reform has been possible due to political support from the higher educational authorities, and private financial support.

CONCLUSIONS

The school mathematics curriculum of Costa Rica is inscribed within important international trends in the teaching and learning of mathematics, which invoke higher-order capabilities as essential factors for teaching and learning in the 21st century. It offers a curricular model that integrates in an original way, knowledge, abilities, higher-order transversal capabilities, problem solving, real contexts, and general mathematical competence. Its implementation has been formally carried out since 2012 through a gradual process, with a variety of documents, digital curricular materials and especially virtual or semi-virtual courses for teachers and students.

In this reform there has been a particular phenomenon, perhaps difficult to replicate in other countries; the strong participation of a team of researchers in mathematics education from public universities at the forefront of both the design and the national implementation of that curriculum. The historical window was opened by a Minister of Education in 2010 and the reform has maintained governmental support from two political parties since that date (something unusual everywhere). Without the (fortuitous) existence of these researchers it would have been impossible for Costa Rica to elaborate this high-quality curriculum in the tight time line that government action required, and the innovative actions that were developed in curricular implementation would also have been impossible. Additionally, decisive financial support for this team has come from Non-Governmental Organizations, and without that support neither the curriculum design nor its implementation would have been developed. Then, an integration of three factors have been the basis of this reform: political will, researchers, private financial support.

The decisions and actions of the reformers have been sustained in what they call a "Perspective of Praxis," which is based on the awareness that the ideas and research condensed in a curriculum – no matter how good they are – only make sense if they materialize in classroom action. This has guided curricular contents, goals, focus, terms, and agendas, timing, activities for the implementation. One example of this is what has happened to assessment, a proposal congruent with the new curriculum was raised by the reformers only in 2017 (Ruiz, 2017). Why? Because before it would have been impossible to reach national consensus and the curriculum official approval could have been compromised.

As has become normal in the world in most national curriculum designs, an important role of technologies is included in Costa Rica both as a vehicle to favor the connection with the generations of students of these times, as well as to promote a "re-engineering" of mathematical objects that acquire different meanings when using technology. However, technology goals are proposed to be developed within the "pedagogical mediation," depending on the many different educational scenarios found in the country. Although there is a strong call for technology use, any mandatory specific curricular objective can be developed without it.

A curriculum radically different from the previous ones demanded the generation of teacher supports (particularly, courses). And the supports could not be limited to traditional face-to-face sessions as had been developed previously (although on a limited basis). Two reasons were decisive. First, the reformers could access the country's teachers with fewer distortions, in a more direct way. The other, everything had to be completed in in the short time remaining in the governmental administration. Both reasons made necessary the intense use of communication technologies. This is why the two-stage process of blended or hybrid courses followed by MOOCs (complemented by a special, more focused, compact and short, personalized modality, referred to as Mini-MOOCs).

Intensive use of digital curriculum resources has underpinned a *Virtual Community of Mathematics Education*. And all this was done considering the conditions in the country and the challenges faced by the curriculum implementation. That is why digital curriculum materials and virtual courses have also been offered for high school students who must prepare for national examinations to complete their high school studies.

In the implementation of the mathematics curriculum in Costa Rica, innovative and high-quality virtual learning environments have been created for both teachers and students. All of these resources should not be seen as cold and aseptic technological developments, but as an expression of the needs of a profound living reform in the teaching and learning of mathematics in a peripheral and developing nation.

It is impossible to ensure that this school reform will have all the desired success, and that it will achieve the proposed aims in the mathematical preparation of Costa Rica's citizens. Much of its success will depend on variables that transcend mathematics and education. And always in this type of countries life transits with more uncertainty than in others. But this experience, developed since 2010, can provide some lessons for the international community.

ACKNOWLEDGEMENTS

This chapter is part of the project "Mathematics Education Reform in Costa Rica," the main agent in the design and implementation of the Costa Rican Math Reform, which is the topic of this chapter. The author is also grateful to Patrick Scott for the language editing of this chapter.

REFERENCES

Barrantes, H., & Ruiz, A. (1995). 1964. In A. Ruiz (Ed.), *Historia de las matemáticas en Costa Rica. Una introducción.* San José, Costa Rica: EUCR, EUNA. Retrieved from http://www.centroedumatematica.com/aruiz/libros/Historia%20de%20las%20matematicas%20en%20Costa%20Rica.pdf

Borba, M. C., Askar, P., Engelbrecht, J, Gadanidis, G., Llinares, S., & Sánchez-Aguilar, M. (2016). Digital technology in mathematics education: Research over the last decade. In G. Kaiser (Ed.), *Proceedings of the 13th International Congress on Mathematical Education, ICME-13 Monographs* (pp. 221–233). doi:10.1007/978-3-319-62597-3_14

Buxton, L. (1981). *Do you panic about maths?* London: Heinemann.

Clarke, D., Emanuelsson, J., Jablonka, E., & Mok, I. (Eds.). (2006). *Making connection: Comparing mathematics classrooms around the world.* Rotterdam, The Netherlands: Sense Publishers.

Clarke, D., Keitel, C., & Shimizu, Y. (Eds.). (2006). *Mathematics classrooms in twelve countries: The insider's perspective.* Rotterdam, The Netherlands: Sense Publishers.

Consejo Superior de Investigaciones Científicas, Spain (CSIC). (2018, January). *Webometrics ranking of world universities*. Retrieved from http://www.webometrics.info/en/Latin_America/Costa%20Rica

Drijvers, P., Barzel, B., Heid, M. K., Cao, Y., & Maschieto, M. (2016). *Uses of technology in lower secondary mathematics education. A concise topical survey.* Cham: Springer. doi:10.1007/978-3-319-33666-4

Fox, A. (2017). Can MOOC and SPOCs help scale residential education while maintaining high quality. In E. Losh (Ed.), *MOOCs and their afterlives* (pp. 37–50), Chicago, IL & London: University of Chicago Press.

Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht: Reidel.

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Dordrecht: Reidel.

Freudenthal, H. (1991). *Revisiting mathematics education: China lectures*. Dordrecht: Kluwer Academic Publishers.

Fullan, M. (2008). *The six secrets of change: What the best leaders do to help their organizations survive and thrive*. San Francisco, CA: John Wiley and Sons, Inc.

Goldin, G. A., Hannula, M. S., Heyd-Metzuyanim, E., Jansen, A., Kaasila, R., Lutovac, S., … Zhang, Q. (2016). *Attitudes, beliefs, motivation and identity in mathematics education. An overview of the field and future directions* (ICME 13 Topical Surveys). Cham: Springer International Publishing.

Hone, K, S., & El Said, G. R. (2016). Exploring the factors affecting MOOC retention: A survey study. *Computers & Education, 98*, 157–168. Retrieved from https://www.sciencedirect.com/science/article/pii/ S0360131516300793

Kilpatrick, J. (1994). Introduction to section 1. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 7–9). Dordrecht: Kluwer Academic Publishers.

Kong, Q. P., Wong, N. Y., & Lam, C. C. (2016). In J. Wang & B. Xu (Eds.), *Trends and challenges in mathematics education* (pp. 27–41). Shanghai: East Normal University Press.

Korhonen, T., & Lavonen, J. (2017). A new wave of learning in Finland: Get started with innovation! In S. Choo, D. Sawch, A. Villanueva, & R. Vinz (Eds.), *Educating for the 21st century* (pp. 447–486). Singapore: Springer.

Jordan, K. (2015, June). Massive open online course completion rates revisited: Assessment, length and attrition. *The International Review of Research in Open and Distributed Learning, 16*(3), 341–358. Retrieved from http://www.irrodl.org/index.php/irrodl/article/view/2112

Leiton, L. (2017, October). Pobreza total de Costa Rica se mantiene y la extrema se reduce. *La Nación*. Retrieved from https://www.nacion.com/economia/indicadores/pobreza-total-de-costa-rica-se-mantiene-y-la-extrema-se-reduce/I6NGELVA6VEIDDY63OCQQVIEYM/story

Leung, A., & Bolite-Frant, J. (2015). Designing mathematical tasks: The role of tools. In A. Watson & M. Ohtani (Eds.), *Task design in mathematics education* (pp. 191–225). New York, NY: Springer.

Liljedalhl, P., Santos-Trigo, M., Malaspina, U., & Bruder, R. (2016). *Problem solving in mathematics education* (ICME-13 Topical Surveys). Cham: Springer International Publishing. doi:10.1007/978-3-319-40730-2_1

Latin American Laboratory for Assessment of the Quality of Education (LLECE). (2014). *Comparison of results between the second and the third regional comparative and explanatory studies. SERCE and TERCE 2006–2013*. Santiago: UNESCO/OREALC.

Maxwell, J. (1989). Mathephobia. In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (pp. 221–226). London: Falmer Press.

Ministry of Public Education (MEP). (2012). *Programas de estudio de Matemáticas para la Educación General Básica y el Ciclo Diversificado*. San José, Costa Rica: Ministry of Public Education of Costa Rica. (Mathematics Programs for Primary and Secondary School). Retrieved from http://www.mep.go.cr/sites/default/files/programadeestudio/programas/matematica.pdf

MOOC Maker. (2016, August 20). *Attrition and retention aspects in MOOC environments* (Working Paper). Erasmus+ Programme of the European Union. Retrieved from http://www.mooc-maker.org/wp-content/files/WPD1.6_INGLES.pdf

Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM Project. In A. Gagatsis & S. Papastavrides (Eds.), *3rd Mediterranean Conference on mathematical education* (pp. 115–124). Athens: Hellenic Mathematical Society.

Niss, M. (2015). Mathematical competencies and PISA. In K. Stacey, & R. Turner (Eds.), *Assessing mathematical literacy* (pp. 35–55). doi:10.1007/978-3-319-10121-7_235

Niss, M., Bruder, R., Planas, N., Turner, R., & Villa-Ochoa, J. A. (2016). Conceptualisation of the role of competencies, knowing and knowledge in mathematics education research. In G. Kaiser (Ed.), *Proceedings of the 13th International Congress on mathematical education* (ICME-13 Monographs, pp. 235–248). doi:10.1007/978-3-319-62597-3_14

Niss, M. (2016). Mathematical standards and curricula under the influence of digital affordances – Different notions, meanings and roles in different parts of the world. In M. Bates & Z. Usiskin (Eds.), *Digital curricula in school mathematics* (pp. 239–250). Charlotte, NC: Information Age Publishing.

OECD. (2010). *PISA 2009 results: What students know and can do – Student performance in reading, mathematics and science* (Vol. I). http://dx.doi.org/10.1787/9789264091450-en

OECD. (2014, February). *PISA 2012 results: What students know and can do – Student performance in mathematics, reading and science* (Volume I, Revised ed.). Paris: PISA, OECD Publishing. http://dx.doi.org/10.1787/9789264208780-en

OECD. (2016). *PISA 2015 assessment and analytical framework: Science, reading, mathematics and financial literacy.* Paris: PISA, OECD Publishing. http://dx.doi.org/10.1787/9789264255425-en

OECD. (2017). *Education in Costa Rica*: *Reviews of national policies for education*. Paris: OECD Publishing. http://dx.doi.org/10.1787/9789264277335-en

Papert, S. (1980). *Mindstorms: Children, computers and powerful ideas.* New York, NY: Basic Books.

Polya, G. (1945). *How to solve it.* Princeton, NJ: Princeton University Press.

Ruiz, A. (1987, October). Fundamentos para una nueva actitud en la enseñanza moderna de las Matemáticas Elementales. *Boletín de la Sociedade Paranaense de Matemática* (Vol. VIII (2)). Curitiba, Brazil.

Ruiz, A. (2000). *El desafío de las Matemáticas*. Heredia, Costa Rica: EUNA. Retrieved from http://www.centroedumatematica.com/wordpress/?page_id=348

Ruiz, A. (2003). *Historia y filosofía de las Matemáticas*. San José, Costa Rica: EUNED. Retrieved from http://www.centroedumatematica.com/aruiz/libros/Historia%20y%20Filosofia/Secciones/Portada.htm

Ruiz, A. (2011, July). La lección a través de estudios comparativos internacionales con videos. *Cuadernos de Investigación y Formación en Educación Matemática* (Number 8). Costa Rica. Retrieved from http://revistas.ucr.ac.cr/index.php/cifem/article/view/6950

Ruiz, A. (2013, July). Reforma de la Educación Matemática en Costa Rica. Perspectiva de la praxis. *Cuadernos de Investigación y Formación en Educación Matemática* (Special Number). Costa Rica. Retrieved from http://revistas.ucr.ac.cr/index.php/cifem/issue/view/1518

Ruiz, A. (2014). Costa Rica: History and perspectives on mathematics and mathematics education. In H. Rosario, P. Scott, & B. Vogeli (Eds.), *Mathematics and its teaching in the Southern Americas* (pp. 250–272). Singapore: World Publishing Co.

Ruiz, A. (2015, April). Balance y perspectivas de la Reforma de la Educación Matemática en Costa Rica. *Cuadernos de Investigación y Formación en Educación Matemática* (Number 13). ISSN 1659-2573. Costa Rica. Retrieved from https://revistas.ucr.ac.cr/index.php/cifem/article/view/19168/19224

Ruiz, A. (Ed.). (2017a). *Teacher preparation in mathematics education in Central America and the Caribbean. The cases of Colombia, Costa Rica, Dominican Republic and Venezuela*. Cham: Springer International Publishing.

Ruiz, A. (2017b, December). Evaluación y pruebas nacionales para un currículo que enfatiza capacidades superiores. *Cuadernos de Investigación y Formación en Educación Matemática* (Special Number). Costa Rica. Retrieved from https://revistas.ucr.ac.cr/index.php/cifem/article/view/31916/31622

Schoenfeld, A. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press.

Schoenfeld, A. (Ed.). (2007). *Assessing mathematical proficiency* (Mathematical Sciences Research Institute Publications, 53). New York, NY: Cambridge University Press.

Shimizu, Y. (2007). What are the characteristics of Japanese lessons emerged by the international comparisons? In M. Isoda, M. Stephens, Y. Ohara, & T. Miyakawa (Eds.), *Japanese lesson study in mathematics* (pp. 68–73). Singapore: World Publishing Co.

Shimizu, Y. (2009). Characterizing exemplary mathematics instruction in Japanese classrooms from the learner's perspective. *ZDM Mathematics Education, 41*(3), 311–318.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1–23.

Suurtamm, C., Thompson, D. R., Kim, R. Y., Díaz-Moreno, L., Sayac, N., Schukajlow, S., … Vos, P. (2016). *Assessment in mathematics education (Large-acale assessment and classroom assessment).* Cham: Springer International Publishing. doi:10.1007/978-3-319-32394-7

Trejos, A. (2018). Why getting rid of Costa Rica's army 70 years ago has been such a success. *USA Today*. Retrieved from https://www.usatoday.com/story/news/world/2018/01/05/costa-rica-celebrate-70-years-no-army/977107001/

UNESCO. (2014). *Global citizenship education. Preparing learners for the challenges of the 21st century*. France: United Nations Educational, Scientific and Cultural Organization. Retrieved from http://unesdoc.unesco.org/images/0022/002277/227729E.pdf

University of Costa Rica (UCR), *School of Mathematics.* (2018). Diagnóstico en Matemáticas. Retrieved from http://www.diagnostico.emate.ucr.ac.cr/

Watson, J. (2008). *Blended learning: The convergence of online and face-to-face education.* Vienna, VA: North American Council for Online Learning. Retrieved from https://files.eric.ed.gov/fulltext/ED509636.pdf

Williams, T., & Williams, K. (2010). Self-efficacy and performance in mathematics: Reciprocal determinism in 33 nations. *Journal of Educational Psychology, 102*(2), 453–466.

Angel Ruiz
Centre for Research in Mathematics and Meta-Mathematics
University of Costa Rica