Chapter 17
Towards a Model for Monitoring and Evaluating Curricula Reforms

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This chapter focuses on general factors that impact on the implementation of curriculum reform programs in the world – some are small scale, targeted interventions while others have national scale and substantial impact; some are located in developing countries, others in more developed countries; some are short-term interventions, others have extended over a long period of time. It is not intended here, therefore, to offer the results of a systematic study on the implementation of curricular reforms around the world, but rather, through some of the experiences or ideas discussed, to identify interesting and relevant dimensions to consider in the processes of implementation of curricular reforms.

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In the first section of this chapter, three national experiences are summarised. These are chosen to provide a variety of elements of curriculum reform programs from which lessons can be learned for the international community. They are chosen with special attention to reform processes in different socioeconomic, geographical and cultural contexts. The criteria for selecting these experiences included: global impact of the reform, diversity between the countries' socioeconomic and cultural development, and relevance to the analytical work we intend to introduce here.

Given the widespread international influence of the Japanese process of Lesson Study (Fernandez & Yoshida, 2012) and the long-established systemic curriculum reform program, Japan was chosen as the first reform experience. In contrast to centralised but often patchily adopted reform efforts in many parts of the world, the Japanese experience provides an example of curriculum reform in a developed country that has become part of the very fabric of the teaching of mathematics. The curriculum intervention in Thailand was chosen as an example of a reform that commenced at a small scale and has rapidly expanded to a much larger scale. A unique feature of the Thailand reform is the use of university agents to implement the reform using a ground-up approach. Costa Rica was chosen as a third reform experience as an example of a wide national-impact process in a developing country. Of particular note is that the reform has so far achieved rare continuity through the support of changing governments.

Together, these three experiences point to some broad dimensions that are considered significant in analysing the impact of curriculum reform implementation internationally. Other reforms will be unpacked in greater detail in the second section of this chapter. Rather than discussing each reform separately, they will be used as exemplars to highlight how particular curriculum reform efforts have taken into account some of the dimensions introduced in the frst section. This will allow discussion of points of convergence or divergence across a larger number of curriculum reform endeavours.

The third section seeks to provide some criteria to assess the development of a curricular implementation. It looks at commonalities and differences across the various reform efforts and at the ways the factors discussed in the previous two sections have affected the impact that each reform has been able to achieve. This section will rely, to a large extent, on anecdotal reflections of the impact of the various reforms, as few have been studied rigorously. We do not, therefore, claim to develop a set of universal criteria to assess the impact of a curriculum reform, but rather seek to identify some pointers arising from the discussions in the first two sections.

Beyond description, we will address the challenge of identifying a model of change, be it explicit or implicit, that underpins the implementation of mathematics curriculum reform. Again, we do not claim universal validity for such a model but offer it as a suggestion that might inform curriculum reform efforts into the future. Together with the discussion in Chaps. 16 and 18, we hope that the lessons learned from discussing the experiences in the implementation of a variety of curriculum reforms will promote more rigorous, systematic and impactful curriculum reform internationally.

Values, Vision, and Goals Within Curriculum-Reform Implementation

The three reform experiences of Japan, Thailand and Costa Rica presented below serve to frame the discussion of factors influencing curriculum reform in the second and third sections. In each case, the reform has meant 'big changes'. However, these changes depended on the realities of those countries; what is to be changed, why it has to be changed, and how it would be changed. Thus, in order to understand the noteworthy success stories of the reform of any country we need to understand the geographical and societal contexts in which the reform is embedded. Japan is a highly developed East Asian country with a strong tradition of centralised curriculum; Thailand is a Southeast Asian country with close links to other ASEAN (Association of South-East Asian Nations) countries; Costa Rica is a rapidly developing Latin-American country.

We draw from these examples three key factors that frame any curriculum reform endeavour: values, vision, and goals. By values, we mean a shared understanding of what is important in the curriculum reform process. By vision we mean the clarity of the intent of the reform. By goals we mean the officially stated goals of the reform. The degree of alignment between the values, vision and goals, the extent to which they match broader societal values and how well they are realised in practice then frame much of the discussion in the remainder of the chapter.

Curriculum Development and Reform in Japan

The Early Years of Curriculum Centralisation and the Beginnings of Lesson Study

Formal education was established in Japan at the university (Daigaku-ryō) in the seventh century CE in order to study written Japanese and arithmetic using Chinese textbooks, including Confucianism. Westernisation of Japanese civilisation[1] and enlightenment began in 1868 after Tokugawa Shogun returned the government to the Emperor, with Japan officially introducing the French public education system up to higher education in 1872. It was an era of educational reform, in which the traditional apprenticeship model moved to whole classroom teaching under a graded curriculum imposed by the government.

[1] Here, the word 'civilisation' does not mean just import Western culture. In the 1860s, the Japanese literacy rate was the highest rate for ordinary people in the world. The International Exposition of Paris (in 1867) became the trigger of Japonism which influenced the European arts, such as Vincent van Gogh and Gustav Klimt, and craftsman industries such as Meissen chinaware. Japanese ethnomathematics was re-developed under the Chinese influence in the sixteenth century and Takakazu Seki and Kanehiro Takebe developed their own original form of calculus in the seventeenth century.

A notable aspect of this early Westernisation was the variety of textbooks adapted from other sources. Despite the introduction of a textbook certification system in 1886 and the introduction of a national curriculum, their content varied. Revising the textbooks through practice then became the custom and gave rise to what has since become known as Japanese Lesson Study. The first theme for lesson study was the improvement of teaching and learning by using Pestalozzi and traditional Zen-Confucian style dialectic methods (Wakabayashi & Shirai, 1883). In 1909, the Elementary School, a laboratory school of the Higher Normal School and the origin of the University of Tsukuba, began to publish the *Journal for Educational Study* to share the themes of Lesson Study for reform. Based on these experiments, the Secondary School proposed a new curriculum for the Ministry in 1910.

In the 1900s, mathematics educators in laboratory schools and the Higher Normal School became aware of and knowledgeable about the Kline movement which aimed to bring different subjects into an integrated mathematics curriculum focused on functional thinking (Isoda, 2019). Despite resistance from some mathematicians, the Ministry promoted the movement by publishing the book *Lehrbuch der Mathematik nach modernen Grundsätzen* (Behrendsen & Götting, 1908) in 1915 and supporting the establishment of the Secondary School Mathematics Society in 1918. In this society, secondary school teachers were able to freely discuss issues of curriculum and pedagogy. In the case of elementary school mathematics, several ideas proposed in the *Journal* and books provided the Lesson Study themes of promoting children as independent learners of mathematics (Isoda, 2007).

Curriculum Development by Teachers and the Evolution of Lesson Study

After World War II, under the government of the United States, the national curriculum was the recommended agenda to enhance school curriculum development. A reform cycle of ten years was established in 1947, with textbooks revised every four years. Curriculum development became the role of every teacher for around ten years, with groups of teachers, educators and mathematicians working to develop curriculum through Lesson Study. One particularly fruitful product of this Lesson Study program was the Japanese didactics of mathematics. This is exemplified in the elementary textbooks developed by the Hiraku Toyama group in the 1960s (Kobayasi, 1989), which have the unique principle of the task sequence moving from the general to the specific. Although these textbooks did not get approval from the government, they were strongly supported by the teachers' union. The union critiqued other approved textbooks.

To address the concerns of the union, educators met the need to systematise terminology in order to more clearly articulate the conceptual sequence in the approved textbooks (Isoda & Nakamura, 2010). The systematised terminology polished theories for: developing mathematical thinking (Isoda & Katagiri, 2012; Katagiri, 1990); designing task sequences (Kobayasi, 1989); representations (Ito, 1971), and; approaches such as open-ended tasks (Shimada, 1977; Becker & Shimada, 1997,

re-theorised by Nohda, 1983). These achievements were published as the guidebooks for Lesson Study. Currently, similar ideas can be seen in the world community such as Iszák and Beckmann (2019): however, Japanese educators have used these theories to develop textbooks and to engage in Lesson Study since the 1960s.

Values, Vision and Goals in Japanese Mathematics

In Japan, curriculum authorities and educators have been working to establish coherence between national curriculum, textbooks and assessments tasks, producing better practices and revision in the reform cycle. National Curriculum reform in Japan has synchronised with Lesson Study, promoting both bottom-up and top-down reform. The national reform committee is selected by the government; at the same time various Lesson Study groups enact objectives of mathematics education through carefully designed task sequences.

A consistent vision has been that educators who enrolled as members of the government committee were to establish consistent improvement of curriculum before and after the US occupation. Development of mathematical thinking and attitude have been consistent aims of Japanese education throughout. Before occupation, developing mathematical and scientific thinking and mathematisation were key directions, while fostering activity and appreciation were reform issues under the US occupation. After the occupation, the first reform in 1956 made mathematical thinking and attitude a key under the scientific and technological necessity for societal development, and in the second reform in 1968, extension and integration became a key under the societal modernisation in which creativity was a necessity.

Curriculum reform and Lesson Study are supported by assessment practices. Since 1956, National Curriculum assessment tests have been used to evaluate the implementation of curriculum. Since 1982, because of teachers' reference to the assessment tasks, this has supported curriculum implementation and reform. Assessment tasks have been revised in order to assess mathematical communication, thinking, and attitude as well as children's achievement up to the junior high schools. Currently, common exam tasks for national universities' entrance at the end of high schools have begun to embed dialectic communication into the exam tasks in order to evaluate students' mathematical thinking.

Despite resistance from some quarters, including some mathematicians and the media, Japanese mathematics education has established goals that emphasise mathematical communication and thinking. The goals are underpinned by values that include teacher participation in bottom-up reform and students as independent learners. Together, the goals and values help to realise a vision of mathematics as a creative and inclusive endeavour essential for the scientific and technological development of society. The development of Lesson Study alongside national curriculum reform has led to a coherence that is rare in international mathematics education.

Mathematics Education Reform in Thailand

The educational reform movement in most ASEAN countries gained traction as the new millennium began. Singapore introduced its 'Thinking School, Learning Nation' program in 1997, and followed this with the 'Teach Less, Learn More' initiative in 2005. These programs aimed to enhance the learning experience for students, promote critical thinking, and allow teachers the opportunity to innovate (MoE, 2013). With particular regard to mathematics, the focus was directed to highlight the process of learning rather than just the content, captured in the pentagon model of curriculum describing skills, concepts, processes, attitudes and metacognition, which has been a feature of Singapore mathematics since 1990 (MoE, 2012). Other ASEAN nations such as Thailand (MoE, 2001), Brunei (Khalid, 2007), and Malaysia (Lim, 2006) followed more recently, adopting a similar direction that is part of a global trend.

Although the content that mathematics students are expected to know and be able to put into practice is well known, it is widely recognised (Inprasitha, 2015; Takahashi, 2015) that in many developing countries the approach to teaching is the area where real innovation is needed. However, successfully implementing reform in the mathematics classroom is particularly difficult as is amply demonstrated by the long journey of reform undertaken by the two most developed countries in the region, Singapore (since the 1970s), and post-war Japan (since 1947). This arduous path has not gone unnoticed by the other countries in the region and they have good reason to be cautious when considering learning transformation in mathematics, which is widely accepted as one of the central pillars of education.

In the case of Thailand, major education reform has followed the global trend exemplified in Singapore. In response to the agenda of the first educational act in 1999, which emphasises *'Reforming Learning Process'* (MoE, 2001; Wasi, 2000), a completely new section, *skills and processes*, was added to the 2001 Basic Education Core Curriculum. Policy makers, curriculum developers, other related educational personnel, and teachers were quick to notice the distinguishing features of this new curriculum, which emphasises not only content or subject matter, but also how students learn best and desirable characteristics to be developed in students (Inprasitha, 2018). Unfortunately, the adoption and implementation of an underlying paradigm shift from a product-oriented approach to a product-process oriented approach in this curriculum reform has not been universal in the broader educational community in Thailand.

To begin to address this, the Faculty of Education at Khon Kaen University in Thailand has undertaken an initiative that gives the university a new and central role in curriculum reform implementation. It has instituted and commenced the 30-year Thailand project (see Fig. 17.1), an attempt to create and incorporate a strong research and development cycle as a system of curriculum and instruction (Inprasitha, in press).

At the commencement of the project, a contextual analysis study was conducted with fifteen student teachers during 2000–2002 to introduce the idea of 'open-ended

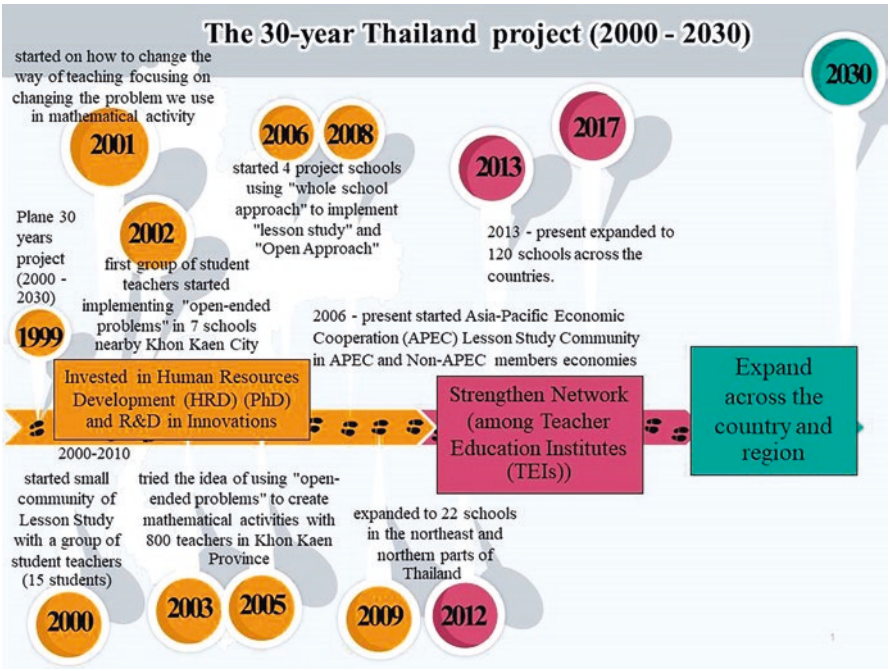

Fig. 17.1 The thirty-year Thailand project

problems' as a part of innovation for teaching mathematics in the collaborative schools in the Khon Kaen city. The Center for Research in Mathematics Education (CRME) was established in 2003 to cultivate a new type of Master's degree program in mathematics education in 2003 and doctoral degree program in 2006. These programs prompted and facilitated professional learning communities among graduate students, teacher educators, mathematics educators, and school principals and teachers.

The role of the graduate students as school co-ordinators, bringing Lesson Study and Open Approach as innovations into schools, is a key initiative aimed at bridging the communication gap between the university and the school. The Open Approach has been adapted by Maitree Inprasitha since 2002 (Inprasitha, 2003) as an innovation for teaching mathematics in Thailand by incorporating three basic steps of Lesson Study (Inprasitha, 2011). The original ideas (Nohda, 2000) are similar to the Open-ended Approach described by Becker and Shimada (1997). Fifth year undergraduate students, trained to use these innovations during the first four years of their teacher initiation program, were sent to schools in 2008.

The first two project schools in 2006 have fully implemented and realised the new section of the 2001 curriculum reform implementation. To institutionalise Lesson Study and Open Approach in the schools, at least three layers of professional learning communities (PLCs) have been created within and among the schools, and in the district (Fig. 17.2). Lesson study teams as members of each PLC

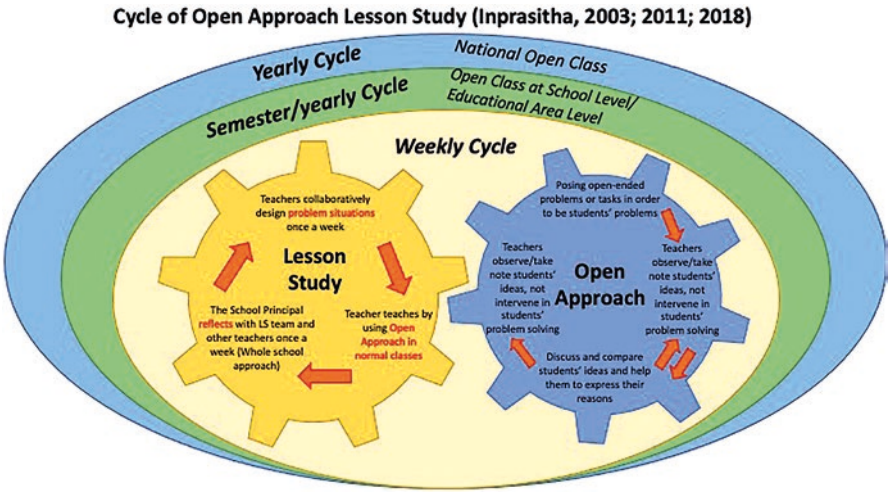

Fig. 17.2 Three layers of professional learning community (PLC)

have been learning together to deeply read the mathematics textbook (translated version) in order to understand new school mathematics; a new teaching approach has been adopted, and; new kinds of assessment have been introduced. This is critical to enable them to faithfully and effectively implement innovations in their schools and in their Lesson Study communities.

During the last twelve years, the Research and Development cycle has been a driving force for curriculum reform implementation with innovations in Thailand and in the region through the Asia Pacific Economic Co-operation Lesson Study[2] project. The first two project schools commenced work in 2006, four schools followed in 2007, twenty-three schools in 2009, which has now increased to nearly two hundred schools in 2018. Approximately fifty Ph.D. candidates and Ph.D. graduates have been working in twenty teacher education institutes across the country.

The Thailand experience shows how a long-term vision, supported at all levels, can grow from small beginnings into a major national reform endeavour. The goals of developing mathematical skills and processes among students are being realised through the agency of PhD candidates and graduates working with teachers to develop a shared vision in Professional Learning Communities. This shared vision is considered essential to the success of this long journey.

[2] See http://www.crme.kku.ac.th/detail_page/Apec2018.html

Mathematics Curriculum Reform in Costa Rica

In Costa Rica, a profound reform of the mathematics curriculum for all primary and secondary education (grades 1–12) began to be gradually implemented in 2013 documented by the Ministry of Public Education (MPE, 2012). A general vision nurtured this reform: It was necessary to respond to decades of curricular backwardness in this school-subject based on up-to-date and appropriate experiences and research from around the world. Global goals were set: to develop higher-order cognitive capabilities across all mathematical areas (to reason and argue, to pose and solve problems, to make connections, etc.), and; to foster a 'mathematical competence' that will enhance understanding and use of mathematics by citizens in diverse contexts. Although there is this a strong emphasis on 'competences', due to local education conditions, the curriculum is based on the mathematical knowledge and abilities that are expected of students (a specific intellectual approach: a curriculum that is neither 'competence-based', nor 'content-based'). Some values were included: an emphasis on real contexts and modelling, as well as the use of technology and mathematics history, are conveyed. Another vision was part of the intellectual foundations: To counterattack 'Mathephobia' (with multiple emphasis or strategies) is a required first aim to achieve learning results, and this nurtures the whole curriculum. To aid these general purposes a specific lesson model for building learning was provided (in other national contexts a model would not be adequate). This model has four steps: problem posing; independent student work; collaborative discussion of strategies, and; closure.

Some international influences can be perceived here: the French Didactique des Mathématiques; the Dutch Realistic Mathematics Education; the NCTM's 'Principles and standards'; the OECD's PISA theoretical framework, and; an *interpretation* of the Japanese Lesson Style. However, as Ruiz (2018) underlines, there are important theoretical roots found in local research developed since the twentieth century. With these visions, values and goals the reformers detached from previous paradigms dominant not only in the teaching of mathematics but in the education establishment itself. It was, using Artigue's (2018) words, a deep "ecological perturbation".

This mathematical reform has had, so far, the support of Ministers of Education of different administrations (2010–2014, 2014–2018, 2018–2022), a political continuity that is rare in Latin American countries. The main means used to design and to guide the implementation of the reform has been the project *Mathematics Education Reform in Costa Rica* (PMERCR), constituted by researchers (specialists in Mathematics Education) from public universities, technology experts and in-service teachers, a team of 12 persons (MPE, 2019a). This combination of professionals has been preserved since the early curricular design. With only the human resources and internal competences within the MPE, it would have been impossible to make progress in the design and implementation of this curriculum.

The researchers and technology experts were funded between 2012 and 2017 by non-governmental organisations; since 2017 the researchers have worked for free

and even self-funded diverse activities and technology-related expenses while the teachers have been supported by the MPE. This fusion of expertise and professional trajectories has allowed a balance for relatively successful curricular design and implementation and has created a bridge between theory and practice. Political continuity did not just happen; it has been carefully cultivated by this team.

Implementation was designed assuming a scenario of changing governments where there would be no continuity in the support. The strategy was to have the greatest possible impact in the shortest period. Here Information and Communication Technology (ICT) was decisive. National "blended-courses" (face-to-face meetings, plus online sessions using Moodle) were carried-out between 2012 and 2016, and fully virtual courses following the MOOC modality began in 2014 (using Class2Go, edX). These courses included a combination of mathematical content with specific pedagogy (all associated with the official curriculum), since, as indicated by Hernández-Solís and Scott (2018), the reformers could not assume that the teachers knew well the mathematics they should teach.

To build the human base that would feed the reform throughout the country, the blended courses were developed in two stages: first, executed directly by the Project's team aimed at teachers and officials who could be leaders; then this group replicated the courses in all regions. Thanks to ICT possibilities, the content, methodologies and assessment were the same in both stages.

After 2017, Mini-MOOCs were built. These constituted an innovative modality with compact courses, each to be completed in less than fifteen hours. MOOCs and Mini-MOOCs were designed not only for teachers but, since 2016, also for high-school students who had to prepare for national exit examinations. The large number of videos that these courses require are directly elaborated, edited by members of the Project.

Since 2019, another type of educational support has been developed: *Mathematics Free Resources* (MFR), open virtual materials aimed at secondary school students without any teacher intervention, though the materials can be used by the latter to design lessons, practices, and assessments (see MPE, 2019b). Most content is developed through videos that should not exceed three minutes. These materials can be accessed through computers, tablets and smartphones. Their use is totally free, no registration process is required. Eventually MFR materials may replace textbooks. The rationale is to focus directly on students due to implementation weaknesses in the classroom or to mitigate eventual socio-political unrest that may limit school activities.

This large amount of high-quality free virtual materials (at the end of 2020: over five hundred web sections, five hundred videos, hundreds of fully explained problems for students and teachers) and actions (multiple courses each year) is a unique experience in Costa Rica (Ruiz, 2020), something that has strongly positioned the mathematics reform in the scenario opened by the Covid-19 pandemic, which obliges educators to adopt a radically different perspective for face-to-face and virtual education working together. This leading-edge role could serve as a key support to sustain this curriculum reform in the years to come.

One of the problems reformers dealt with was how to incorporate curricular objects in the task design, classroom actions and assessment, especially higher-order capabilities (processes) and levels of complexity. That is why Ruiz (2018) elaborated a new theoretical framework for task-design that can be used in the preparation of lessons, assessment, and national high-stakes testing. This framework includes a model with sixty-one precise indicators to identify and gauge in three levels the participation of the five higher-order capabilities, or processes, of the curriculum in a mathematical task. This facilitates the determination of the level of complexity of any mathematical task and the conditions for its use in the classroom and in all educational dimensions. This intellectual framework, though not official, goes further than the curriculum approved in 2012.

The Math Reformers in Costa Rica have thus generated a large amount of multiple innovative resources, professional development has been provided for many teachers, and teacher preparation programs at the public universities are synchronised with the new curriculum. However, the curriculum implementation has progressed unevenly. Programa Estado de la Nación (PEN, 2017, 2019) reports feeble use in the classrooms of the four-steps model and the Problem-Solving strategy. Ruiz (2018) points out a weak introduction of higher-order capabilities in the classroom actions, assessment, and national examinations, and also that official documentation and guidelines are not fully consistent with the mathematics curriculum, weakening its implementation.

Reasons for the uneven implementation include:

- An ideological one: in the minds of educational agents and in official documents, behaviouristic paradigms (or curricular views reduced to contents, no abilities, or higher-order capabilities) still dominate;
- The weak preparation of teachers, most of whom come from private universities of dubious quality (a country with just over 5 000 000 inhabitants has more than 50 private universities);
- An inadequate system of teacher recruitment and professional development that is not based on teacher quality performance;
- An inefficient classroom management and teaching system, including:
    - teaching work loads of 30 to 32 hours per week all of which are student contact;
    - overload of administrative tasks assigned to teachers;
    - weak academic use of time in the classroom;
    - feeble advising-supervising-monitoring of classroom action (what happens in a classroom is almost a ‘black-box’), and;
- There has been always resistance from some higher-level and regional officials within the Ministry of Education to implement this curriculum.

These general conditions impact differently on the national regions that have unequal socio-economic and cultural environments, common in most developing countries.

From its inception, it was clear that success in such a wide and deep reform would take 25 to 30 years, depending on factors within mathematics and also on others that would transcend it. It was a bold decision that however would have consequences. As Artigue (2018) has emphasised, a curriculum reform invokes unpredictability; but it will be even more unpredictable if it means a profound 'ecological perturbation' and implies a long-term implementation. For example, here, at least six government transitions will be implied as well as the need for a sustained investment of resources. A 'point of no-return' will never be insured. The situation becomes more uncertain with the general weakening of education processes due to the Covid-19 pandemic.

Some elements to underline include:

- *politics*: taking advantage of a historical 'window' and cultivating support from diverse social-political agents;
- *resources*: with the best international standards, but 'tailored' to the national reality and curriculum implementation; and
- *ICT*: intensive, innovative utilisation.

There was also a central *implementation vision*: Curriculum design should not be done "in vitro" accompanied afterwards by implementation actions; implementation needs to be part of the design from its inception. This vision is what Ruiz (2013) termed a "Perspective of praxis in mathematics education".

One relevant and important feature is the existence and continuity of a team with strong expertise that assumed the mathematical reform as a national and personal commitment. This has secured the permanence of coherent visions, values and goals. This is not easy to replicate, but it may be noteworthy for curricular implementation in developing countries.

What Factors Intervene in the Implementation of Reformed Curricula?

The preceding section presented examples of three different curricular reforms in three different contexts. Together these examples point to some factors that are important to consider when designing or evaluating the implementation of curricular reforms in different contexts. Some of these factors are *external* to the curricular reform, others are *internal*, and others relate to *realisation*. By external we mean those factors that are located beyond the reform itself – these may be international influences, geographic challenges or the political and societal context in which the reform takes place. By internal we mean those factors that are part of the reform itself – these may include the development processes in the reform, the emphases within the reform or the target audience of the reform. By realisation we mean the resources developed as part of the reform, the role of assessment in the reform or professional work with teachers as part of the reform.

For example, Japanese reform is influenced by external factors such as strong cultural traditions of collaboration between teachers and researchers and a school context where education is highly valued, and by internal factors such as clear protocols for planning and implementing curriculum supported by thoroughly documented resources. The curriculum reform in Thailand is strongly influenced by external factors, such as the Southeast Asian context, the influence of Japanese theories and the geographic challenges of implementing the reform in a large developing country, but it is equally influenced by internal factors, such as the role of the university and its post-graduate students in promoting the reform.

In the case of Costa Rica, the ongoing reform is influenced by external factors such as the political context of changing governments and the uneven, often poor preparation and professional development of teachers but a generally agreed need to develop more literate and informed citizens; it is also influenced by internal factors such as the collaboration between researchers from universities, technology experts and teachers and the development of virtual resources (MOOCs, Mini MOOCs, MFR) to implement and support the reform.

This sub-section expands on and elaborates the external, internal and realisation factors considered important in planning and evaluating curriculum reforms. The factors described were identified inductively from the papers and presentation of the various curriculum reform programs during the ICMI Study conference. Critical factors in each reform were identified and summarised, from which key factors in the implementation of mathematics curriculum reform were identified. These are presented below.

Description of General Factors

External Factors

1. International influences
   (a) In what way is the reform implementation influenced by international trends or processes?
   (b) How and to what extent is it influenced by international comparisons of student achievement?
2. Geographical influences and reach
   (a) What is the scale of the reform? Is it localised, regional or national?
   (b) Are there particular geographic challenges that need to be considered?
3. Political influences and ownership
   (a) Does the societal and political context within which the reform takes place impose particular imperatives that need to be addressed in its implementation?

(b) Who has ownership of the reform? Is it centralised or devolved?
(c) To what extent is the reform influenced or supported by the general community?

4. Time-scale

(a) Does the reform have long or short-term goals? How does this influence the implementation of the reform?

Internal Factors

5. Development processes

(a) Who worked on the curriculum development and its implementation? Was it top-down, bottom-up or some combination? How does this impact on ownership of the implementation of the reform?
(b) How much time was invested?

6. Emphases in the curriculum itself

(a) What is the balance between skills and content?
(b) What cognitive competences are emphasised?
(c) What is the role of digital technologies?
(d) How do these factors impact on the implementation of the reform?

7. Target audience

(a) Is the reform for everyone or a particular target group?

Realisation Factors

8. Resources

(a) What resources are provided? What is their role?
(b) Who develops the resources?
(c) Are the resources coherent and in line with the intended curriculum reform?

9. Teachers

(a) What guidance is provided for teachers?
(b) How much autonomy do teachers have in implementing the reform?

10. Assessment

(a) Is the assessment aligned with the reform goals?
(b) What is the role or influence of assessment in the implementation of the reform?

How These Factors Intervene in Implementation of the Reform

External Factors

In addition to the examples provided above, the experiences described in papers relating to curriculum reform in England and Mexico (Lozano et al., 2018) and Luxembourg (Nadimi & Siry, 2018) give somewhat contrasting examples of how international trends have impacted on the implementation of curriculum reform projects.

Lozano et al. (2018) compare and contrast curriculum reform initiatives currently taking place in Mexico and England. In both countries the curriculum reform represents a radical break from existing practice, casting teachers as agents and innovators of curriculum reform rather than as mere implementers. In each case the reforms were at least partly a response to perceived failings of current practice reflected in scores on international assessments of student achievement, and in the case of England the reform was strongly influenced by international practice such as that found in East Asia.

In both cases, resources and texts were produced which challenged existing practice, giving explicit pedagogical guidance to teachers regarding representations and strategies for calculations. The resources emphasise conceptual coherence and understanding, providing innovative approaches to the teaching of concepts. Rather than being concerned with the fidelity of teachers' implementation of curriculum interventions, consistent with the East Asian approach teachers in both countries are offered the opportunity to make decisions based on insights derived from research and practice made explicit within the materials. Although the reforms are works in progress, early indications suggest that the reforms are beginning to transform teaching and learning by re-imagining teachers and curriculum designers as partners in innovation.

Nadimi and Siry (2018) provide a very different example of an historical curriculum in Luxembourg reform strongly influenced by international emphases, but ultimately of limited impact on promoting curriculum reform. Structural reforms addressing the entire school system were proposed in 1958 with the goal of linking all levels of schooling and linking school more closely to active citizenship. Public perceptions were that school was neither preparing Luxembourgian students adequately for further studies in neighbouring countries nor proving useful for developing informed citizens. However, implementation was hampered by external factors including the school system itself and language. A structural reform of secondary schools in 1968, which removed the differentiation between boys' and girls' experiences in school mathematics, provided the impetus for bringing together classic and modern mathematics and emphasising practical applications as well as abstract concepts.

In contrast, attempts to reform primary school mathematics were hampered by challenges such as language. As the language of instruction in Luxembourgian primary schools is German, it was not possible to import Belgian texts that were

written in French, and it took several years before Luxembourgian texts including modern mathematics were developed. In short, the reform was not adapted to the existing culture of the school system in Luxembourg and failed to achieve its intended goal of radically reforming mathematics education. Nevertheless, it did provoke discussions about school mathematics in Luxembourg, helping to unify mathematics education for boys and girls and to revise and modernise the applications of school mathematics.

In considering how external factors impact on the implementation of mathematics curriculum reform it is also important to take account of the proposed scale of the reform and whether the reform reaches its target audience. Two contrasting examples are provided by the experience of developing a national curriculum in Australia (Sullivan, 2018) and reforming senior secondary mathematics for non-academic students, i.e. those not intending to study high level mathematics at tertiary level, in Israel (Karsenty, 2018).

After a number of relatively unsuccessful attempts to introduce a more coherent national approach to schooling in Australia, the Australian Curriculum, Assessment and Reporting Authority was constituted in 2008 to develop a national curriculum for Foundation (the year before school) to Year 10. The intent was to improve the quality, equity and transparency of Australia's education system. School education in Australia, however, is constitutionally the responsibility of eight state and territory governments, hence the development of a national curriculum caused a blurring of the lines of responsibility. The result is arguably at best a compromise position in which the national curriculum has been agreed upon in principle yet interpreted and implemented differently across the nation.

In his paper, Sullivan (2018) describes how this differential interpretation has limited the extent to which the underpinning philosophy of the national curriculum is realised in practice. While the scope of the curriculum reform in Australia was national in intent, the political context of eight different states and territories each having ultimate responsibility for curriculum implementation meant that compromises were made and that some of the ideals espoused in the national curriculum have not yet been realised in practice.

In contrast to the centralised national reform described above that was compromised, at least to some degree, by regional interests, Karsenty (2018) describes how the 3 U reform in Israel commenced as a pilot in two schools and is gradually extending. The reform was designed for low-track students in the senior high school years, commencing with an extensive phase of research-based design of new learning materials coupled with an extensive model of teacher support and dissemination. The issue of students' experience of long-term failure in mathematics was tackled head-on through the development of resources that engaged students' common sense and real-life experiences, made extensive and integrated use of a variety of visual and other representations and minimised technical manipulations and notation.

Teachers were introduced to the materials through workshops and summer courses and invited to participate in school-based trials. There was initial reluctance from many teachers based on claims about the limited capacity of students, limited

time and the effort required. These concerns informed an extensive program of on-site, ongoing support provided to those teachers who agreed to trial the resources, the success of which has led to the expansion of the program from an initial cohort of two schools, six teachers and a hundred students to thirty-two schools, one hundred and ninety-one teachers and four thousand, seven hundred and fifty students.

Together these contrasting examples show the importance of carefully considering factors such as geographic reach, political influence and ownership. The Australian national curriculum reform impacted significantly on existing state-based curriculum and was effective in stimulating a national debate about priorities in school mathematics. Yet political control in the various states and territories limited the extent to which the lofty intentions of the national reform were implemented in practice. The 3 U curriculum reform in Israel was much more modest in both its target audience and geographic scope, yet the extensive program of school-based support generated a level of ownership among teachers that has led to a significant expansion to, and implementation by, a much wider group of schools.

A final external factor that is important to consider in curriculum implementation is the time-scale of the reform. Short-term acceptance of curriculum reform programs is perhaps the norm; long-term sustainability is rare (Schoenfeld, 2006). Lyle, Cunningham and Gray (2014), for example, in their examination of one school's work in implementing the Australian national curriculum point to the negative impact of "change fatigue" arising from frequent top-down changes in policy. In contrast the contextual and tailored professional learning solutions and respectful support of the 3 U curriculum reform in Israel described above has enabled it to continue for some fifteen years.

The Thailand reform described in the first section is a particularly significant and promising initiative that takes a long-term view of change. Rather than expecting large numbers of teachers to make rapid and dramatic changes in practice, the Thai reform adopts a 30-year implementation timeframe, commencing with post-graduate students as agents of change whose influence will gradually permeate the entire country.

Internal Factors

A key issue in any curriculum reform is the development process. Regardless of whether a reform is top-down or bottom-up, every reform poses its own set of challenges. In particular, the development process impacts strongly on ownership of the reform and hence on its implementation. The Luxembourg and Australian curriculum projects discussed above were very much top-down processes with the inherent challenges of gaining traction among teachers suffering change fatigue. The 3 U curriculum project was much more bottom-up, not seeking to change the existing curriculum but seeking to develop resources and work with teachers to improve outcomes for disinclined students. The challenge here was to achieve reach among a wider group of schools.

An alternative in which top-down and bottom-up processes work together is described in the first section. The project 'Mathematics education reform in Costa Rica' was developed through a collaborative effort of researchers and technology experts from public universities funded by non-government organisations and in-service teachers allocated by the Ministry of Public Education. This enabled the project to develop a balanced approach to curriculum design and implementation, bridging theory and practice and cultivating political continuity. While success will take many years, the project is addressing key social, economic and educational issues. The existence and continuity of a team with strong expertise required considerable effort and commitment but serves as a model for the implementation of other national curriculum reform projects, particularly in developing countries.

Every curriculum reform has particular emphases built into it. Many, if not all, involve increased attention to the skills and cognitive competences required for active citizenship in an increasingly technological environment. In some cases, this has meant a corresponding de-emphasis on traditional mathematics content. This raises challenges for the implementation of the reform, particularly when the emphases clash with existing practice.

Tran, Nguyen, Nguyen, Ta and Nguyen (2018) describe a teacher preparation project in Vietnam developed in response to curriculum reform emphasising mathematical modelling as one of five competences including communication, mathematising, reasoning and argument, solving problems and using mathematical tools. Historically the curriculum and texts in Vietnam have made little connection between mathematics and the real world, hence the reform curriculum represents a radical change. The project described by Tran et al. seeks to develop increased mathematical literacy and modelling skills among preservice teachers as agents of change in the Vietnam education system.

The project seeks to investigate effective processes to prepare preservice teachers to teach mathematics contextually and to document the influences, successes and failures of the implementation on preservice teachers' knowledge and practice. The preservice teachers expanded their knowledge and appreciation of mathematics as much more than a set of isolated skills or concepts. Rather, the skills and competences of mathematical literacy were developed alongside knowledge such as linear programming and regression analysis. Although this is a small-scale project in one university, it holds promise for the implementation of the mathematical modelling reform more widely, as the teachers become agents of change in the Vietnam education system.

Changsri (2018) describes a similar shift in teacher perceptions among preservice teachers in Thailand who engaged in a process of Lesson Study and open approach to problem solving using videos that were part of the APEC Lesson Study project. The emphasis on student thinking challenged preservice teachers' beliefs about mathematics, moving away from traditional content with right or wrong answers towards valuing processes and students' ideas through real-world problems. The perceived role of the teacher changed from one of imparter of knowledge and judge of correctness of answers to one of problem poser, listener and prompter of thinking. Again, the small-scale project positions preservice teachers as

agents of change in the wider implementation of Lesson Study and the open approach in Thailand.

Every curriculum reform has a specific target audience. In some cases, such as the Costa Rican reform it may be the entire national cohort of students; in others such as the reform in England described by Coles (in Lozano et al., 2018) it may be students from particular year levels; in the 3 U reform in Israel it was a cohort of low achieving students in the senior secondary years. Each approach brings its own implementation challenges and opportunities that need to be addressed in appropriate ways. The experience of a STEM-focused project stimulated by one enthusiastic and knowledgeable teacher in one Hong Kong school (Mok & Sung, 2018) provides an interesting counterpoint to many of the larger scale projects. A three-year enrichment program for talented students, led by the teacher, was progressively introduced by the school to promote communication, analysing and problem-solving skills.

Evaluation of the program indicated that the students in the high ability group developed higher academic achievement, higher order thinking and greater self-esteem. A key to the success of the program was the experimental approach used to design and refine the lessons with careful application of relevant learning theories. Citing Cai and colleagues (2017), Mok and Sung conclude that a key to successful reform implementation is to develop and test learning sequences at a grain size that is useful to teachers. We suggest that many of the issues experienced in the implementation of large-scale curriculum reforms are related to grain size – focusing only on macro-questions of curriculum or textbook design may ignore the day-to-day realities of the teachers responsible for its implementation.

Realisation Factors

The remaining three factors, resources, teachers and assessment, relate to the realisation of the reform. They are discussed in greater detail in Chap. 18. Here we touch briefly on the importance of these realisation factors to help frame the consideration of curriculum reform success discussed in the next section.

Resources play a key role in each of the three case studies discussed in the section "Description of general factors". In the case of Japan, the development of consistent textbooks and assessment practices built on Lesson Study have been instrumental in establishing lasting reform; in the case of Thailand, postgraduates have been key resources as agents of change in the system; in the case of Costa Rica, the development of MOOCs and Mathematics Free Resources has helped circumvent political, teacher quality and geographic issues. Similarly, Lozano and colleagues (2018) highlight the central role played by textbooks that provide pedagogical advice to teachers alongside content, Karsenty (2018) discusses the development of resources aimed at underachieving senior secondary students that present relevant and engaging real life problems, while Changsri (2018) discusses the value of lesson videos as a tool to stimulate preservice teachers' analysis and reflection in a Lesson Study approach.

While resources were instrumental in the implementation of these reform programs, Rodríguez-Muñiz, Díaz and Muñiz-Rodríguez (2018) describe how resources that do not align well with curriculum priorities and emphases can equally limit the impact of a reform. They describe how new secondary curricular learning standards in Spain aim to promote a less formal approach to statistics and probability and focus more on applying mathematics to social science contexts. Statistical literacy, the integration of technology into mathematics and context-based problem-solving involving estimation, simulations and conjectures are key aspects of these standards.

Yet an examination of five full series of textbooks revealed that with one exception every example referred to quantitative rather than qualitative variables, references to variability were extremely rare and more than 95% of exercises were algorithmic in nature. Similarly, probability questions were based on laws of counting with no reference to subjective probability. In this way the textbooks maintained the focus of previous curriculum standards, being an inhibitor rather than a promoter of the changes recommended in the curriculum standards.

An interesting and unusual interpretation of what constitutes a 'resource' is discussed in the description of the Australian reSolve: Mathematics by Inquiry project (Thornton et al., 2018). A key aspect of this project is the recruitment and professional development of 300 Champion teachers whom the authors considered to be not only implementers of the reform but part of the project resources. Many of these teachers were involved in the development of the material resources of the project, which, similar to those described in the UK and Mexico resources, are intended to be educative in nature (Davis & Krajcik, 2015). Thornton, Tripet and Patel argue that resources and documentation alone seldom produce sustainable change, even when accompanied by professional learning to promote implementation. In contrast the reSolve project aims to position the three hundred Champions as part of the project resources, integrally intertwined with the material resources through the project philosophy.

Considering teachers as resources in the implementation of curriculum reform is therefore critical in ensuring uptake. In the South African context, Brodie (2018) describes the development of professional learning communities (PLCs) in the Data-Informed Practice Improvement Project (DIPIP). School-based professional learning communities were supported to participate in a sequence of developmental activities analysing learners' errors in different contexts. They engaged in activities such as test analysis, learner interviews, concept analysis and planning, as well as videoing and reflecting on lessons. The project produced substantial and sustained improvements among teachers in each of the three professional learning communities.

An analysis of the conversations in the PLCs showed an increase in conversations focused on student learning and thinking and highlighted that the focus on pedagogical content knowledge supported teachers to work on their content knowledge. Brodie concludes by arguing that a model of extended inquiry in PLCs, focusing on both knowledge and practice, can be a powerful way of encouraging

responsiveness to learners, increasing teachers' professional agency and accountability and hence contributing strongly to the implementation of the reform.

Student errors and learning is also the focus of a curriculum project in Italy that takes advantage of large-scale standardised tests of achievement (Martignone et al., 2018). Rather than seeing the national INVALSI[3] (*Istituto Nazionale per la Valutazione del Sistema Educativo di Istruzione e di Formazione*) tests as a means for comparing schools or groups of students, the researchers worked with groups of teachers to compare statistical data about one's own classes with that of the school or of the population more broadly to identify specific strengths and weaknesses.

This can contribute to the work of curriculum implementation consistent with the goals of the intended curriculum, enabling teachers to reflect on the relationship among the intended, implemented and attained curriculum. Martignone, Ferretti and Lemmo suggest that an analysis of test tasks can thus be used as a tool to modify the system itself and carry key messages about its implementation. However, as discussed in Chap. 18, assessment practices that are not aligned with the curriculum reform may serve to at least partially derail the reform.

This section has identified and synthesised a number of external, internal and realisation factors impacting on the implementation of mathematics curriculum reform. Illustrative examples have been provided as a means of elaborating those factors. This is by no means a complete list of potential factors, nor is it intended to be an in-depth analysis or discussion of the curriculum projects described in the papers and presentations. We hope, rather, that this section sets the scene for the following discussion of the assessment of the success of curricular reform.

The Assessment of Curricular Reform Success

The factors considered in the previous subsection point the way to the possibility of identifying criteria that allow us to evaluate progress or lack of it in an implementation experience. Of course, every curriculum reform has some successes and some failures. It is not our intent, therefore, to attempt to provide a definitive process through which a curriculum reform can be evaluated, but rather to suggest how those responsible for the introduction of a curriculum reform might reflect on the experience. In addition, it is our hope that the discussion might promote systematic evaluation as an integral part of the curriculum reform process rather than an add-on.

Building on the discussion in the previous section, we suggest that three fundamental qualities should be considered in evaluating a curriculum reform: external *cohesion*, internal *coherence* and realisation *fidelity*.

[3] INVALSI is the Italian National Institute for the Evaluation of the Educational System of Education and Training. http://www.invalsi.it/invalsi/istituto.php?page=chisiamo

Relationship and Alignment Between Curriculum Reform Factors

External Cohesion

As discussed above every mathematics curriculum reform takes place within a national, educational and cultural context. Reform that ignores, or worse contradicts the conditions in which it is located is therefore likely to be at best short-lived. The New Math reform in Luxembourg, or indeed in Western society more generally (e.g. Kilpatrick, 2012) provides an example of a reform that failed to take account of at least some external factors. New Math was stimulated by a political context in which strong mathematics education was seen as essential to combating the perceived threat that Western countries such as the USA would fall behind in the international technology race, yet it failed to take into account the educational context in which it was introduced. Teachers were generally unprepared for the radical shifts in emphasis in the curriculum and other important elements of mathematics often described as basic skills were marginalised. In the case of Luxembourg (Nadimi & Siry, 2018) the school system itself was unable to respond to the demands of the new curriculum.

A key element of external cohesion is therefore support at every relevant level. This includes support of the educational authorities involved, support of general academic agents such as mathematical societies, science councils, education boards and universities, support of school related educational agents such as advisors, supervisors, principals, wider support of politicians and the general public, and essentially support of teachers themselves. For large-scale reforms such as those described in Costa Rica, Thailand and the UK and Mexico, it is critical that such support is evident at all levels; for smaller-scale reforms such as those described in Israel or South Africa, gaining the support of those involved is likely to depend at least partly on the extent to which the reform is consistent with external factors such as national priorities and directions.

We therefore suggest two critical implementation questions related to external cohesion:

1. To what degree has the reform been able to gain the support of:
   - teachers and others responsible for its implementation;
   - mathematicians, mathematics educators and mathematical or mathematics education groups, councils or societies;
   - educational and curriculum authorities and unions;
   - politicians, the media and the general public?
2. To what degree has the reform been able to sustain support over time?

Internal Coherence

In the first section of this chapter we identified vision, values and goals as key aspects of any curriculum reform. We described the degree to which vision, values and goals are aligned in practice as internal coherence. Of course, the vision, values and goals are strongly influenced by external factors but once articulated they become a material part of the curriculum reform and its documentation. Successful curriculum reform requires that all elements of, and actors in, the curriculum and its implementation have a shared view of the vision, values and goals. This includes their articulation through tasks or statements of content and proficiencies, assessment and crucially programs of professional learning.

Internal coherence, such as that found in the Japanese approach to Lesson Study and accompanying texts and the Singapore curriculum founded on the pentagon model and implemented in a national system of education in which research, professional learning and preservice teacher education all work in the same direction, is likely to lead to continuity over time. On the other hand, rapid changes in priorities work against the production of a set of shared values, a shared vision and shared goals across the elements of the reform.

In many Western countries, the drivers of recent curriculum reforms have had a political dimension, with an accompanying move away from competences associated with high level mathematical thinking and problem solving and back to facts and content. Most often this has been motivated by perceived poor student performance in international measures of assessment. This has left teachers in an ideological and practical dilemma: on the one hand research and their own experience point to progressive, student-centred and open approaches, on the other hand the political imperative points to more closed, transmissionist and content-focused approaches. Lack of shared vison, values and goals puts any curriculum reform in jeopardy, leading to teacher burn-out and change fatigue and ultimately de-professionalising and disempowering those who are central to the educational endeavour.

We therefore suggest two critical implementation questions relating to internal coherence:

1. To what degree does the reform exhibit coherence and continuity of values, visions and goals associated with:
   - mathematics itself, as a discipline in its own right, as a subject essential for technological and scientific advancement and as a key element of active and informed citizenship;
   - mathematical education, the pedagogical approaches and priorities recommended;
   - assessment of mathematical learning, both at a system level and at an individual school and teacher level?
2. How well are the values, vision and goals communicated in the wider community?

Realisation Fidelity

No matter how well a curriculum reform addresses and is sensitive to the contextual factors in which it is located, nor how consistently the values, vision and goals are documented nor how well they are communicated to key agents of reform, the danger exists that they may not be exhibited in practice. As Schoenfeld (2006) declared, "Indeed, one can imagine curricular materials that, when used in the way intended by the designers, result in significant increases in student performance, but, when used by teachers not invested or trained in the curriculum, result in significant decreases in student performance" (p. 17). That is, the resources developed must not only be faithful to the values, visions and goals of the reform, but as discussed by Mok and Sung (2018) they must also speak to teachers at an appropriate grain-size that enables them to be implemented in practice. Similarly, professional learning and assessment must be both faithful to the values, vision and goals of the reform and have practical impact for teachers and preservice teachers at all levels.

We therefore suggest three critical implementation questions relating to realisation fidelity.

1. To what degree do the resources developed in the reform enable agents to faithfully implement the crucial aspects of the reform?
2. To what extent do professional learning and preservice teacher education programs position teachers as co-designers and agents of reform?
3. Is large and small-scale assessment integral to the reform and aligned with the values, vision and goals of the reform?

A Proposed Model Describing the Relationships Between Factors in Mathematics Curriculum Reform Implementation

Drawing on the lessons from the experiences described in the first section, on the discussion of factors impacting on curriculum reform in in the second section and on the questions regarding evaluation of curriculum reform suggested in this section, we now propose a model describing the relationships between factors in mathematics curriculum reform that might help to inform the planning, implementation and evaluation of reform initiatives. As before, we do not claim completeness nor universal applicability. However, we hope that the model will provide a point of reference for governments, educational systems, universities and for schools as they seek to enhance mathematics education.

Rather than being linear in nature, the model is reflexive and dynamic, recognising that all elements in the curriculum reform interact and influence each other. In this way a curriculum reform is a complex dynamic system in which the factors involved in its design and implementation are far from settled when the reform makes its way into the education system through the official adoption of texts or documents.

The proposed model is presented in Fig. 17.3.

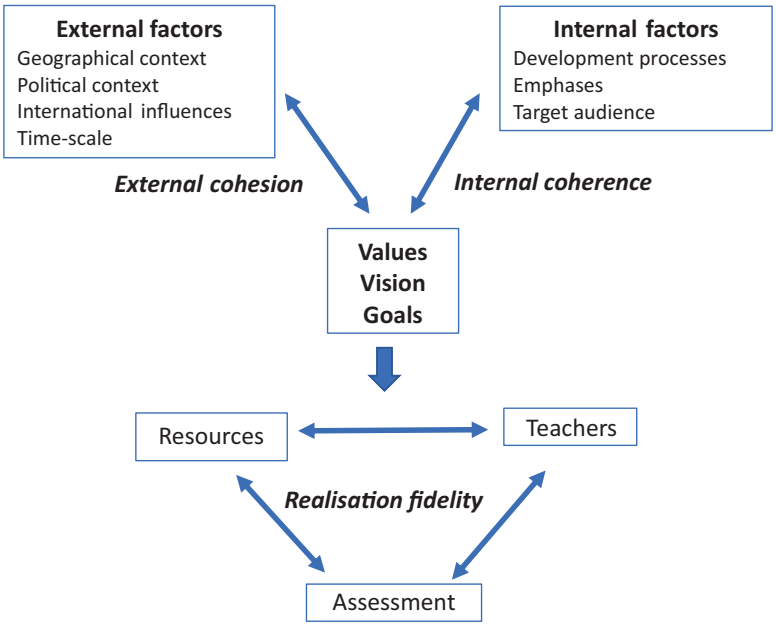

Fig. 17.3 A proposed model describing the relationships between factors in mathematics curriculum reform implementation

Conclusion

This chapter has examined factors associated with the implementation of mathematics curriculum reform. We have identified a number of internal, external and realisation factors. These factors have helped inform questions about internal coherence, external cohesion and realisation fidelity that are important elements to be considered in the evaluation of the implementation of mathematics curriculum reform. Finally, they have helped to suggest a model in which the external, internal and realisation factors interact as parts of a complex dynamic system (Fig. 17.3).

As we have pointed out, the discussion is neither complete nor definitive. Few, if any, of the reforms described in this chapter have been rigorously or systematically evaluated. For this reason, we have avoided labelling them as 'successful' or 'unsuccessful' as every reform has its positive and negative aspects. However, what we can assert from the case studies in the first section and the specific illustrations of the factors discussed in the second, is that unless a curriculum reform works coherently across the external, internal and realisation dimensions, its implementation is likely to be problematic.

We note also that every reform takes place in a particular cultural and political context, among a particular target audience and at a particular scale. Hence the relative importance of the factors identified in this chapter will be specific to the context. For these reasons, we caution against the wholesale importing of a curriculum

initiative from one context to another. However, we hope that we have been able to point to some factors that will allow curriculum developers to undertake a systematic and well-considered approach to the planning, implementation and evaluation of a mathematics curriculum reform initiative.

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